1.1 Algebraic methods

- To solve an inequality involving algebraic fractions:
  - Step 1: multiply by an expression squared to remove fractions
  - Step 2: rearrange the inequality to get 0 on one side
  - Step 3: find critical values
  - Step 4: use a sketch to identify the correct intervals

分数形式的 inequality

ex. $\frac{x^2}{x-3} < -x+1$

所以画图，穿针引线法，从右上开始，从右往左穿。

1.2 Using graphs to solve inequalities

画图方法：如何求渐进线？（ex. $y=\frac{7x}{3x+1}$）

竖直渐进线：分母为0（$3x+1=0$）

水平渐进线：最高次系数比（分子分母次数相同）

分母次项高于分子次项渐进线不存在

分子次项高于分母 用极值来做 ex. $y=\frac{4x^3+7}{3x^2+2}$　　$y=\frac{4x+\frac{7}{x^2}}{3+\frac{2}{x^2}}=0$

所以此类情况渐进线为 $y=0$

1.3 Modulus inequalities

⊗ 算出来 CV，要画图看是否相交

题型1：ex. $|x^2-4x|<3$.

step 1：画图　step 2：判断交点时图像的函数

做法2：两边同时平方，绝对值变为正.

记绝对值画图法.

$y=\sin|x|$，图像关于 y 轴对称

$y=|\sin x|$，x 轴下方翻到上去.

Chapter 2. Series

如果范围为多少不行

2.1 The method of differences 差值法

$$\sum_{r=1}^{n} u_r = \sum_{r=1}^{n}\left(f(r)-f(r+1)\right) = f(1)-f(n+1)$$

目的：变成减法，更好求解总合值.

题型(a)：a express ...... in partial fractions.

题型：(b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

b Consider $\sum_{r=1}^{n}\left(r^2(r+1)^2-(r-1)^2r^2\right)$

Let
- $r=1$: $\cancel{1^2(2)^2} - (0)^2 1^2$
- $r=2$: $\cancel{2^2(3)^2} - \cancel{(1)^2 2^2}$
- $r=3$: $\cancel{3^2(4)^2} - \cancel{(2)^2 3^2}$
- $\vdots$
- $r=n$: $n^2(n+1)^2 - \cancel{(n-1)^2 n^2}$

有可能取多个 $r=n-1$ 和 $r=n-2$.

Sum of terms $= n^2(n+1)^2$

Then $4\sum_{r=1}^{n} r^3 = n^2(n+1)^2$

So $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

公式补充（一般会给）

$$\sum_{k=1}^{n} c = cn$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

相加　$S_n = na_1+\frac{n(n-1)}{2}d$　$S_n=\frac{n(a_1+a_n)}{2}$　　乘　$\frac{a_1(1-r^n)}{1-r}$

Chapter 3 Complex Numbers.

3.1 Exponential form of complex numbers

补充公式

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

Modulus – argument form of a complex number.

$z = r(\cos\theta + i\sin\theta)$

推导

Euler's relation

$e^{i\theta} = \cos\theta + i\sin\theta.$

θ超范围要减

$\boxed{z = re^{i\theta}}$ $r = |z|$ $\theta = \arg z.$

Example 题型

$z = 2e^{[illegible]}$，导致超出范围，所以要减到范围以内.

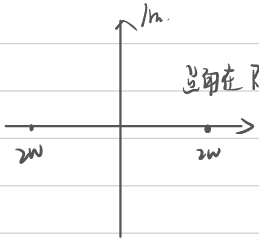
3.2 Multiplying and dividing complex number.

- $|z_1 z_2| = |z_1||z_2|$
- $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
- $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

→可用于求 z^2

- $z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$ – $\frac{z_1}{z_2} = \cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)$

Ex6 $\text{Im}(zw) = 0$ →



总角在Re轴上

思路：先求总角，再求单角，相减得另一角

补充：归纳法 induction

step1：证明 $n=1$ 成立 step 4 结论

step2：Assume $n=k$ 成立.

step3：在step2的基础上证明 $n=k+1$ 成立

3.3 De Moivre's theorem

$(r(\cos\theta + i\sin\theta))^n = r^n(\cos n\theta + i\sin n\theta)$

补充：real and positive 没有复数值

3.4 Trigonometric identities

Links $(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n, n \in \mathbb{N}$

where ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ ← **Pure 2 Section 4.3**

题型: Express $\cos^n\theta$ or $\sin^n\theta$

$(\cos\theta + i\sin\theta)^6 = \cos 6\theta + i\sin 6\theta$ ⟶ 思路: ① 用二项式定理展开次方式 (得次方幂), 摘出 cos 项

② 倒公式 (关于 cos 单项公式) — 得 $\cos n\theta$ or $\sin n\theta$ 的幂

两种方法结果不一样

$$\frac{1}{z} = z^{-1} = (\cos\theta + i\sin\theta)^{-1}$$
$$= (\cos(-\theta) + i\sin(-\theta))$$
$$= \cos\theta - i\sin\theta$$

- $z + \frac{1}{z} = 2\cos\theta$
- $z^n + \frac{1}{z^n} = 2\cos n\theta$
- $z - \frac{1}{z} = 2i\sin\theta$
- $z^n - \frac{1}{z^n} = 2i\sin n\theta$

$z^n + \frac{1}{z^n}$ 与 $(z+\frac{1}{z})^n$ 不一样!!!

$2\sin n\theta$ $\quad 2^n\sin n\theta$

3.5 nth roots of a complex number

$z = r(\cos\theta + i\sin$

$z^n = w$, 几次方就代表它有多少个解.

算解时, θ 的取值为 $[-\pi, \pi]$

- For any complex number $z = r(\cos\theta + i\sin\theta)$, you can write $z = r(\cos(\theta + 2k\pi) + i\sin(\theta + 2k\pi))$, where k is any integer.

$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$

$\sin(a-b) = \sin(a)\cos(b) - \sin(a)\cos(b)$

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$

$\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan a\tan b}$

$\tan(a-b) = \frac{\tan(a)-\tan(b)}{1+\tan a\tan b}$

解题思路: $z = r(\cos(\theta+2k\pi) + i\sin(\theta+2k\pi))$

θ 可求, 把 w^n 的表达式写成 modulus-argument form.

$n\theta_? = \theta + 2k\pi$

画圆轮用

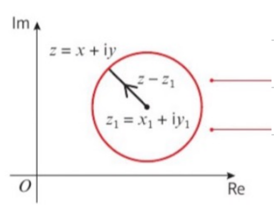
$\cos(5\pi - 6\alpha)$
$= -\cos 6\alpha$
$\cos(2\pi - 4\alpha)$
$\cos(4\alpha)$

- $1, \omega, \omega^2, \ldots, \omega^{n-1}$ form the **vertices** of a regular n-gon
- $1 + \omega + \omega^2 + \ldots + \omega^{n-1} = 0$ $\quad\omega$ 为新解

Watch out Make sure you choose n **consecutive** values of k to get n distinct roots. If an argument is not in the interval $[-\pi, \pi]$ you can add or subtract a multiple of 2π.

4.1 loci in an Argand diagram

- For two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, $|z_2 - z_1|$ represents the distance between the points z_1 and z_2 on an Argand diagram.



Transfor to Cartesian form

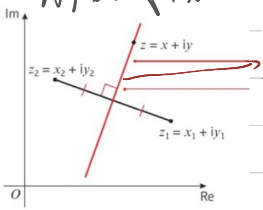
强题型减法

$$|z - z_1| = r$$
$$|(x - x_1) + \mathrm{i}(y - y_1)| = r$$
$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

Since $|p + q\mathrm{i}| = \sqrt{p^2 + q^2}$

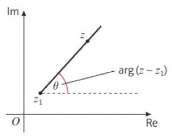
角平分线情况

- Given $z_1 = x_1 + \mathrm{i}y_1$ and $z_2 = x_2 + \mathrm{i}y_2$, the locus of points z on an Argand diagram such that $|z - z_1| = |z - z_2|$ is the perpendicular bisector of the line segment joining z_1 and z_2.



轨迹就是一条线

A half-line



Cartesian equation

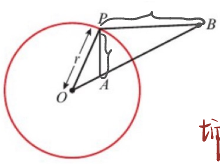
$$\arg(z - z_1) = \theta$$
$$\arg((x - x_1) + \mathrm{i}(y - y_1)) = \theta$$
$$\frac{y - y_1}{x - x_1} = \tan\theta$$
$$y - y_1 = \tan\theta(x - x_1)$$

4.2 Further loci in an Argand diagram

找到两固定点(以 constant ratio) 的 loci

解题思路：平方找 cartesian equation

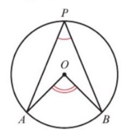
$BP = kAP$



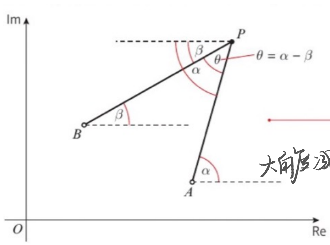
$|z - a| = k|z - b|$ (轨迹符号)

tips：因为要满足模型，必须一个点在圆内，一个点在圆外

- **The angle subtended at the centre of the circle is twice the angle at the circumference.**



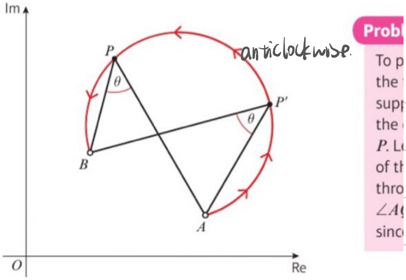
$\angle AOB = 2\angle APB$



大角度减小角度

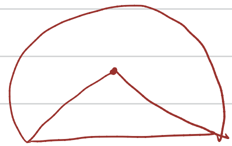
$$\angle APB = \alpha - \beta$$
$$\theta = \alpha - \beta$$
$$= \arg(z - a) - \arg(z - b)$$ ☆
$$= \arg\left(\frac{z - a}{z - b}\right)$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$$



题型：1 sketch the locus.
2. find the Cartesian equation (图形法)

算



求面积 需要用到
扇形面积公式
$S=\frac{1}{2}\theta r^2$

If $\theta < \frac{\pi}{2}$, then the locus is a **major arc** of the circle.

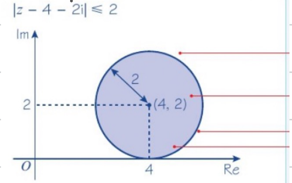
If $\theta > \frac{\pi}{2}$, then the locus is a **minor arc** of the circle.

If $\theta = \frac{\pi}{2}$, then the locus is a semicircle.

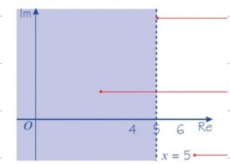
4.3 Regions in an Argand diagram

三种类型

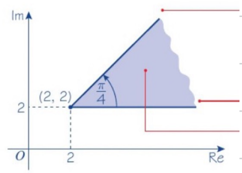
$|z-4-2i| \leqslant 2$



ii $|z-4| < |z-6|$



i $0 \leqslant \arg(z-2-2i) \leqslant \frac{\pi}{4}$



4.4 Further regions in an Argand diagram

三种基本类型的结合

格式：$\{z \in \mathbb{C} : \sim\} \cap \{z \in \mathbb{C} : \sim\}$

4.5 Transformations of complex plane

做题技巧：把 z 用 w 表示

- $w = z + a + ib$ represents a translation by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a, b \in \mathbb{R}$.
- $w = kz$, where $k \in \mathbb{R}$, represents an enlargement by scale factor k with centre (0, 0), where $k \in \mathbb{R}$.
- $w = iz$ represents an anticlockwise rotation through $\frac{\pi}{2}$ about the origin.

ex:
$x^2+y^2=4$ 可换成 $|z|=2$.

当问到 z lies on the real axis in the z-plane，这时需实虚部分开

利用 $(a-b)(a+b)=a^2-b^2$
底数为正.

$$z = \frac{(u+2)+iv}{u+i(v+1)}$$

$$z = \frac{(u+2)+iv}{u+i(v+1)} \times \frac{u-i(v+1)}{u-i(v+1)}$$

Chapter 5 First-order differential equations

5.1 First-order differential equations with separable variables

$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$ 解出来后还要加 C

C 有很多不同的值 → family of solution curves

5.2 First-order linear differential equations of the form $\frac{dy}{dx} + Py = Q$.

$$\boxed{f(x)\frac{dy}{dx} + f'(x)y = \frac{d(f(x)y)}{dx}}$$

$$e^{\int P dx}\frac{dy}{dx} + e^{\int P dx}\cdot Py = e^{\int P dx}\cdot Q$$

√ 推导过程.

Multiply the equation by the integrating factor f(x).

Then $f(x)\frac{dy}{dx} + f(x)Py = f(x)Q$ **(1)**

The equation is now exact and so the left-hand side is of the form

$f(x)\frac{dy}{dx} + f'(x)y$

So $f(x)\frac{dy}{dx} + f(x)Py = f(x)\frac{dy}{dx} + f'(x)y$

$\therefore\quad f'(x) = f(x)P$

Dividing by f(x) and integrating

$$\int \frac{f'(x)}{f(x)}\,dx = \int P\,dx$$

$\therefore\quad \ln|f(x)| = \int P\,dx$

$\therefore\quad f(x) = e^{\int P dx}$

Equation **(1)** becomes

$e^{\int P dx}\frac{dy}{dx} + e^{\int P dx}Py = e^{\int P dx}Q$

$\therefore\quad \frac{d}{dx}\left(e^{\int P dx}y\right) = e^{\int P dx}Q$

$\therefore\quad e^{\int P dx}y = \int e^{\int P dx}Q\,dx + C$

5.3 Reducible (可还原) first-order differential equations

题目类型：给 $\frac{dy}{dx} = \sim$ ，给 $u/z = f(y)$

步骤 $\frac{dy}{dx}$ 替换 $\frac{dy}{dx} = \frac{dz}{dx} \sim$

转换成关于 x 与其他变量的式子

for example, $\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$

Chapter 6 second-order differential equations

6.1 Second-order homogeneous differential equation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

步骤 1: A.E (Auxiliary equation) $am^2 + bm + c = 0$

算出根进行判断 (α 与 β) G.S 之形式

Case 1: $b^2 > 4ac$ 有两实根

$$y = Ae^{\alpha x} + Be^{\beta x}$$

Case 2: $b^2 = 4ac$ 有相等实根

$$y = (A + Bx)e^{\alpha x}$$

Case 3: 有 two imaginary roots $\alpha, \beta = \pm i\omega$

$$y = A\cos\omega x + B\sin\omega x$$

Case 4: $p \pm qi$

$$y = e^{px}(A\cos qx + B\sin qx)$$

6.2 Second-order non-homogeneous differential equations

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

Find the general solution to the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{2x}$

As in Example 8, the complementary function is $y = Ae^{3x} + Be^{2x}$.
The particular integral cannot be λe^{2x}, as this is part of the complementary function.

Watch out The fu
C.F. and satisfies th
$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

解题步骤: ① 用 $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy$ 算出 C.F (complementary function) (也就是 G.S)

② find P.I (particular integral) 看 f(x)

Form of f(x)	Form of particular integral
p	λ
$p + qx$	$\lambda + \mu x$
$p + qx + rx^2$	$\lambda + \mu x + \nu x^2$
pe^{kx}	λe^{kx}
$p\cos\omega x + q\sin\omega x$	$\lambda\cos\omega x + \mu\sin\omega x$

一定要代进原式看是否合理

Form of f(x)	Form of P.I.
k	a
kx	$ax + b$
kx^2	$ax^2 + bx + c$
ke^{px}	λe^{px}
$m\cos\omega x$	$\lambda\cos\omega x + \mu\sin\omega x$
$n\sin\omega x$	$\lambda\cos\omega x + \mu\sin\omega x$
$m\cos\omega x + n\sin\omega x$	$\lambda\cos\omega x + \mu\sin\omega x$

$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 3$

First consider the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 0$
$m^2 - 2m = 0$
$m(m - 2) = 0$
$\Rightarrow m = 0$ or $m = 2$
So the complementary function is $y = A + Be^{2x}$.
The particular integral cannot be a constant, as this is part of the complementary function, so let $y = \lambda x$.

③ 用 P.I 和公式 算 $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ 代入原式 算 P.I 中常数

④ G.S. $y = \text{C.F.} + \text{P.I.}$

λxe^x is part of complementary function.

E/P 5 a Explain why λxe^x is not a suitable form for the particular integral for the differential equation
$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$ **(2 marks)**

b Find the value of λ for which λx^2e^x is a particular integral for the differential equation. **(5 marks)**

6.3 Using boundary conditions (find particular solution)

一般会给 $\frac{dy}{dx}$ 和 y 的值 在同一 x 值下

① 先求 G.S ② 用 G.S 代入求解

6.4 Reducible second-order differential equations

目标：把 $\frac{dy}{dx}$ 和 $\frac{d^2y}{dx^2}$ 用 $\frac{dy}{dt}$ 表示

Chapter 7 Maclaurin and Taylor series

7.1 higher derivates

Ex 3. $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_r x^r$ 推导 Maclaurin 公式

$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \dots r a_r x^{r-1}$

$f''(x) = 1 \times 2 a_2 + 3 \times 2 a_3 x + \dots r(r-1) a_r \cdot x^{r-2}$

$f'''(x) = 3 \times 2 \times 1 a_3 + \dots r(r-1)(r-2) a_r x^{r-3}$

$f(0) = a_0 \quad f'(0) = a_1 \quad f''(0) = 1 \times 2 a_2 \quad f'''(0) = 3 \times 2 \times 1 a_3$

$a_2 = \frac{f''(0)}{2!} \qquad a_3 = \frac{f'''(0)}{3!}$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots \frac{f^r(0)}{r!}x^r$$

Maclaurin series

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(r)}(0)}{r!}x^r + \dots$$

only true if the given series converges 收敛

倒数时那部分会用到.

Maclaurin polynomial of degree 1 是写到 x 的一次项

$\sin 10^\circ$ 代入要转换成 $\frac{\pi}{18}$

Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + \left(\frac{n(n-1)\dots(n-r+1)}{r!}\right)x^r + \dots$$

- The expansion is valid when $|x| < 1$, $n \in \mathbb{R}$

7.3 Series expansions of compound functions

- $e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots$ for all x
- $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1}\frac{x^r}{r} + \dots$ $\quad -1 < x \leq 1$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots$ for all x
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots$ for all x
- $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots$ $\quad -1 \leq x \leq 1$

5 a $\ln(1+x-2x^2) = \underline{\ln(1-x)(1+2x) = \ln(1-x) + \ln(1+2x)}$

$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots, \qquad -1 \leq x < 1$

For example.

$1+x \quad (1-3x) \rightarrow x$ 替换成 $-3x$

$\cos x$ 中的 x ~~密~~ $\boxed{替换}$ 成 $2x^2$.

$$\ln\left(\frac{\sqrt{(1+2x)}}{1-3x}\right) = \ln\sqrt{1+2x} - \ln(1-3x)$$

$$= \frac{1}{2}\ln(1+2x) - \ln(1-3x)$$

Example 8:

show that $e^{\sin x} \approx 1 + x + \frac{x^2}{2} - \frac{x^4}{8}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

$e^{x - \frac{x^3}{3!} + \frac{x^5}{5!}} = \left(1 + x + \frac{x^2}{2} + \cdots\right)\left(1 + \left(-\frac{x^3}{6}\right) + \cdots\right)$ 把 x 替代成 $-\frac{x^3}{6}$

7.4 Taylor series

$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots + \frac{f^{(r)}(a)}{r!}(x-a)^r + \cdots$

题目形式 is powers of $(x+4)$

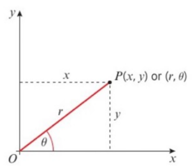
7.5 Series solutions of differential equations

$\frac{dy}{dx}$ 一般给出, $\frac{d^2y}{dx^2}$ 要代入 x, y 求解, $\frac{dy}{dx}$ 再求导求 $\frac{d^3y}{dx^3}$

Chapter 8 Polar coordinates

8.1 Polar coordinates and equations.

polar coordinates are written as (r, θ)



convert between Cartesian { coordinates and polar coordinates
equation (通常是和) 由 x, y 组成

- $r\cos\theta = x$
  $r\sin\theta = y$
- $r^2 = x^2 + y^2$
  $\theta = \arctan\left(\frac{y}{x}\right)$

例: polar equation 形式: r 关于 θ 的方程

$x = \frac{2a\sqrt{2}}{3} \times \cos\alpha = \frac{2a\sqrt{2}}{3} \times \frac{\sqrt{2}}{\sqrt{3}} = \frac{4a}{3\sqrt{3}}$

So the equation of the tangent is:

$r = \frac{4a}{3\sqrt{3}}\sec\theta$

10 a
$r^2 = \sec 2\theta$
$r^2\cos 2\theta = 1$
$r^2(2\cos^2\theta - 1) = 1$
$2r^2\cos^2\theta = 1 + r^2$
$2x^2 = 1 + x^2 + y^2$
$\therefore \quad y^2 = x^2 - 1$

b
$r^2 = \operatorname{cosec} 2\theta$
$\Rightarrow \quad r^2\sin 2\theta = 1$
$\Rightarrow \quad 2r\sin\theta\, r\cos\theta = 1$
$\Rightarrow \quad 2xy = 1$
$y = \frac{1}{2x}$

8.2 Sketching curve.

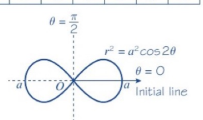
基础图形 记住形状

- $r = a$ is a circle with centre O and radius a.
- $\theta = \alpha$ is a half-line through O and making an angle α with the initial line.
- $r = a\theta$ is a spiral starting at O.

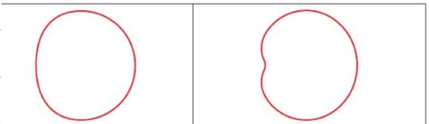
c $r^2 = a^2\cos 2\theta$

You need values of θ in the ranges $-\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$ and $\frac{3\pi}{4} \leqslant \theta \leqslant \frac{5\pi}{4}$ 要先找出一个 loop 的范围, 在范围里描点作图 用 when r=0 求解

θ	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$
r	0	a	0	0	a	0



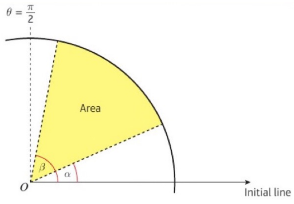
$r = a(p + q\cos\theta)$ 公式有固定的形状



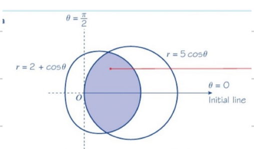
Example 8 通过 Argand diagram 找 polar equation，实质就是 Argand ⇒ Cartesian → polar 的转换

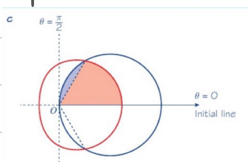
8.3 Area enclosed by a polar curve

- The area of a sector bounded by a polar curve and the half-lines $\theta = \alpha$ and $\theta = \beta$, where θ is in radians, is given by the formula
  Area = $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$



如果问一个 loop 的 area，要找出 loop 的范围





8.4 Tangents to polar curves

$x = f(\theta)\cos\theta$

$y = f(\theta)\sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

To find a tangent parallel to the initial line set $\frac{dy}{d\theta} = 0$

To find a tangent perpendicular to the initial line set $\frac{dx}{d\theta} = 0$