Homework 3
Due Wednesday, February 17

1. Consider the following Landau functional:

$$S[\boldsymbol{\phi}] = \int_\Lambda d^d x \left\{ \frac{1}{2} \sum_i \left[(\boldsymbol{\nabla}\phi_i)^2 + r\phi_i^2 \right] + u(\sum_i \phi_i^2)^2 + v \sum_i \phi_i^4 \right\}.$$

   Here ϕ_i, $i = 1, \ldots, n$ is an n-component real field. This Landau functional describes critical properties of a generalized Heisenberg ferromagnet (the first three terms) with a cubic anisotropy (last term). Cubic anisotropy will arise in any magnetic material with cubic symmetry due to spin-orbit interactions, which will tend to align spins with the crystal axes. The usual Heisenberg ferromagnet would have $n = 3$, here you will consider a generalized model with an n-component order parameter.

   (a) Following the same steps as in the case of the one-component ϕ^4 theory (corresponding to an Ising magnet), discussed in class, derive RG flow equations for this model (show all details):

$$\begin{aligned}
\frac{dr}{d\ell} &= 2r + 4\frac{S_d}{(2\pi)^d}\frac{\Lambda^d}{r+\Lambda^2}[(n+2)u + 3v], \\
\frac{du}{d\ell} &= (4-d)u - 4\frac{S_d}{(2\pi)^d}\frac{\Lambda^d}{(r+\Lambda^2)^2}\left[(n+8)u^2 + 6uv\right], \\
\frac{dv}{d\ell} &= (4-d)v - 4\frac{S_d}{(2\pi)^d}\frac{\Lambda^d}{(r+\Lambda^2)^2}\left[12uv + 9v^2\right].
\end{aligned}$$

   (b) Find all fixed points (you should find 4 of them) and analyze their stability for $d < 4$ as a function of n. Note that for $n < 4$ the only stable fixed point has $v^* = 0$. This means that the critical point has a full rotational symmetry, i.e. the cubic anisotropy is suppressed by fluctuations at the critical point. This is an example of an *emergent symmetry*, i.e. symmetry which is not present in the microscopic system and is generated by fluctuations.

   (c) Analyze the RG flows for $n < 4$ and $n > 4$ in the $u - v$ plane and find the critical exponents corresponding to all stable fixed points.