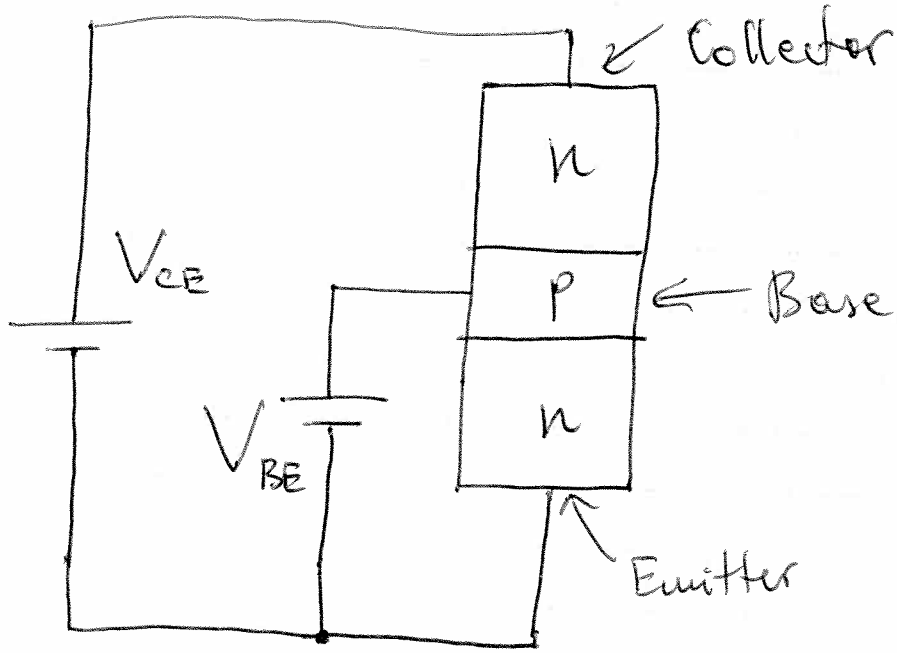
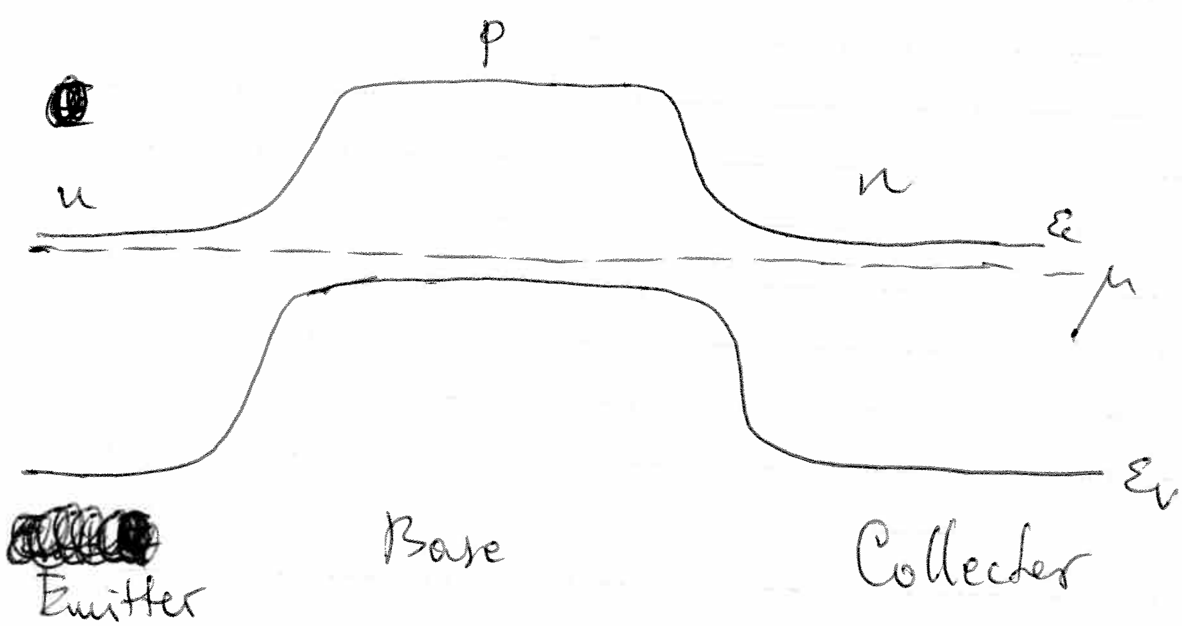
lecture 22

Continue semiconductor devices...

Bipolar transistor – involves two p-n junctions.

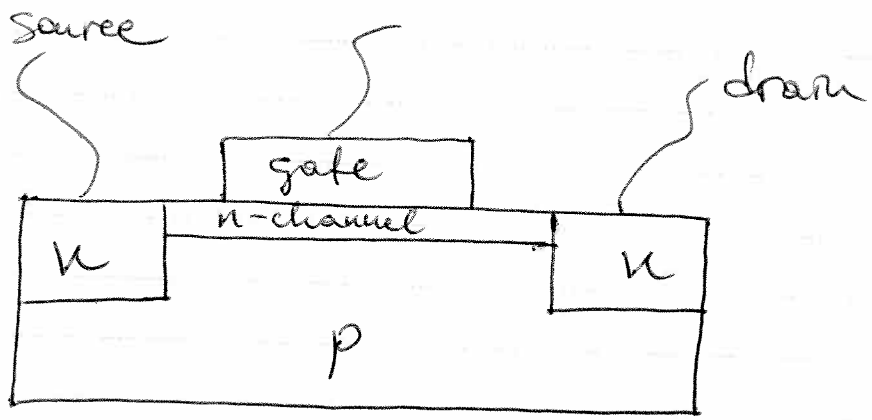




Small changes in V_{BE} lead to huge changes in the current flowing in the CE circuit.

· Transistor can work either as a switch or as an amplifier.

More modern type of transistor - field-effect transistor (FET).



Gate voltage opens or closes n-channel and controls the source-drain current.

Heterojunctions

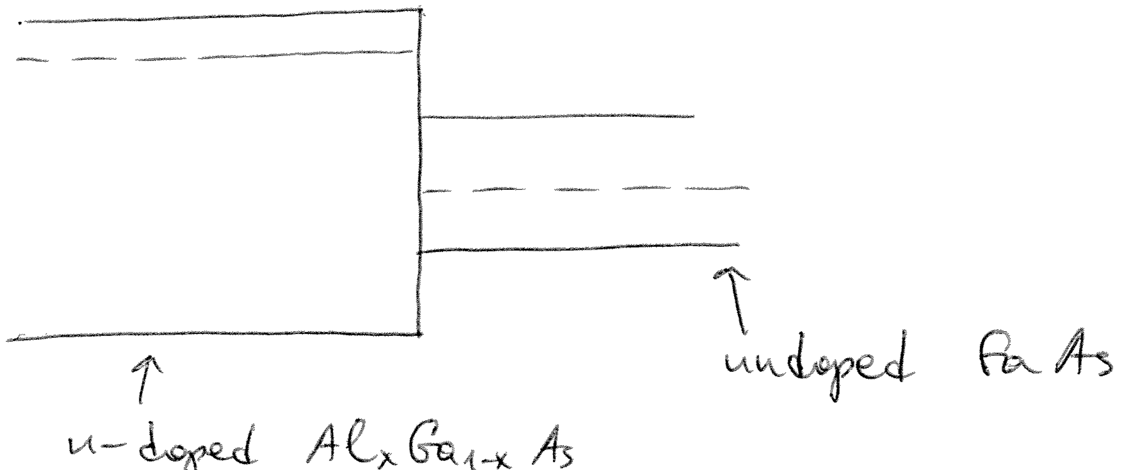
Heterojunctions - junctions of different semiconductors, instead of the same semiconductor with different doping levels.

Typical heterostructure: AlGaAs - GaAs.

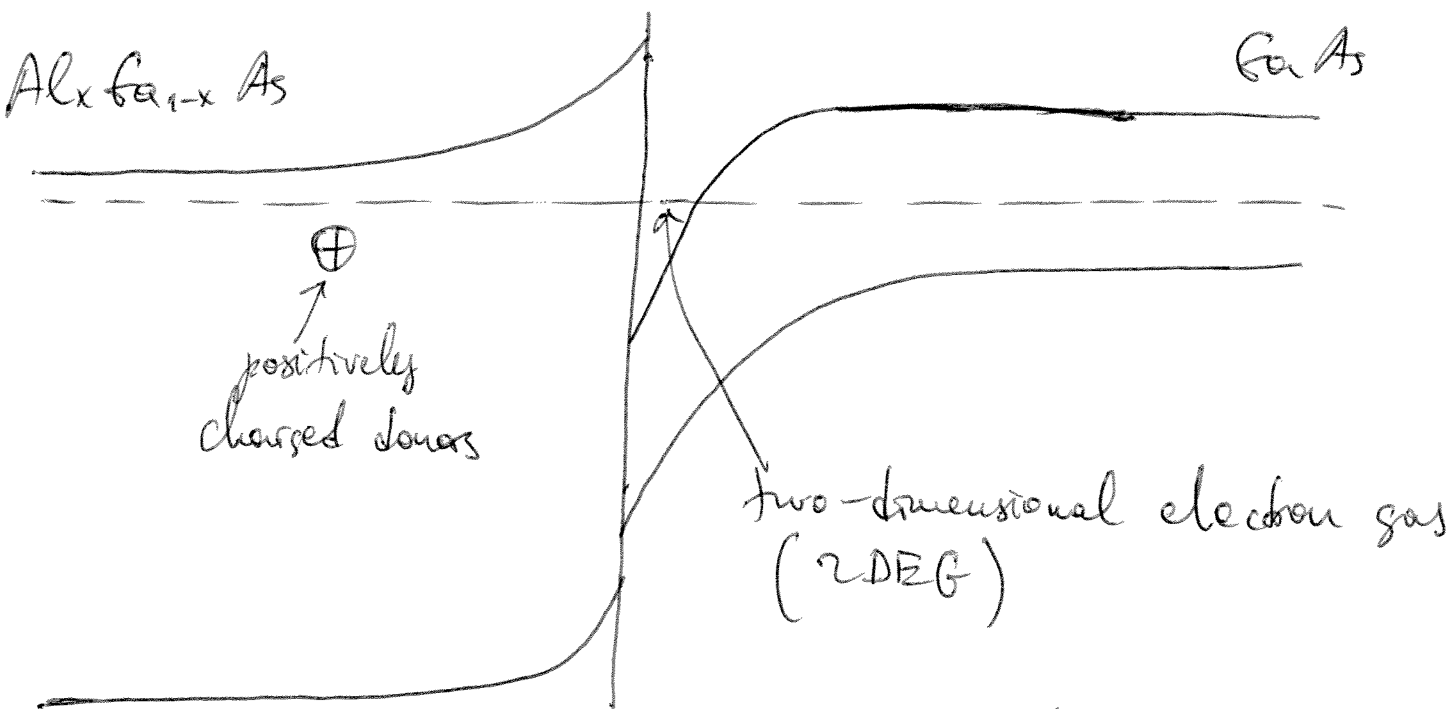
GaAs : $E_{gap} = 1.42\,eV$

AlAs : $E_{gap} = 2.16\,eV$

Alloy $Al_x Ga_{1-x} As$ - the gap varies smoothly between AlAs and GaAs.
AlAs and GaAs have nearly the same lattice constant and same crystal structure - easy to put them together.



Electrons from AlGaAs will start diffusing into GaAs, leaving positively charged donors behind.



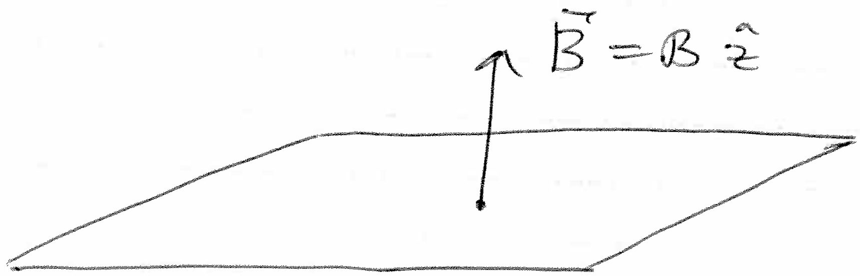
Doping is typically done as thin doped layer, separated from the interface with GaAs. This separation minimizes scattering of the electrons in 2DEG on the potential of positively charged donors $\Rightarrow$ can achieve very high electron mobilities: $\sigma = \frac{ne^2\tau}{m^*}$, $\mu = \frac{e\tau}{m^*}$ - mobility.

- Modern high-mobility transistors are based on heterojunctions.

Ability to create ultraclean 2DEG lead in the 1980's to the discovery of the Quantum Hall Effect.

Quantum Hall Effect

Consider 2DEG in a perpendicular magnetic field.



So far we have been treating the effect of magnetic field on electrons semiclassically:

$$\hbar \frac{d\vec{k}}{dt} = -\frac{e}{c}\vec{v} \times \vec{B}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{\hbar \vec{k}}{m^*}$$

This gives:

$$\frac{d^2 x}{dt^2} = -\frac{eB}{m^* c}\frac{dy}{dt}$$

$$\frac{d^2 y}{dt^2} = \frac{eB}{m^* c}\frac{dx}{dt}$$

General solution corresponds to motion on a circle of arbitrary radius:

$$\vec{r} = R\left[\cos(\omega_c t + \delta), \sin(\omega_c t + \delta)\right]$$

$$\omega_c = \frac{eB}{m^* c}$$ – cyclotron frequency.

Semiclassical treatment is OK provided:

$\omega_c \tau \ll 1$ – this condition means that electron can't complete a full circle before getting scattered by impurity.

If $\omega_c \tau > 1$, electrons move around circular orbits ⟹ quantum mechanics becomes important.

In clean 2DEG in magnetic field can't treat magnetic field semiclassically – need full quantum mechanical solution.

The Hamiltonian of an electron in magnetic field has the form:

$$H = \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A}\right)^2 - \vec{\mu}\cdot\vec{B}$$

$\vec{B} = \vec{\nabla} \times \vec{A}$, $\vec{A}$ is the vector potential.

$\vec{\mu} = -\mu_B \vec{S}$ – magnetic moment of the electron.

$\mu_B = \frac{e\hbar}{2mc}$ – Bohr magneton.

Neglect the $\vec{\mu} \cdot \vec{B}$ term for now.

To see why the Hamiltonian has this form, recall classical mechanics of a charged particle in magnetic field.

$$m\vec{v} = \vec{p} - \frac{q}{c}\vec{A} = \vec{p} + \frac{e}{c}\vec{A}$$

Here $m\vec{v}$ is the mechanical momentum:

$$E_{kin} = \frac{m\vec{v}^2}{2}$$

$\vec{p}$ – canonical momentum.

Correspondence principle: replace canonical momentum by the quantum-mechanical momentum operator:

$$\vec{p} = -i\hbar\vec{\nabla}.$$

Vector potential $\vec{A}$ is defined up to a gauge ~~[illegible]~~ transformation:

$\vec{A} \to \vec{A} + \vec{\nabla} f$ doesn't change $\vec{B}$ since $\vec{\nabla} \times (\vec{\nabla} f) = 0$

Thus there are many different forms (gauges) one can choose for $\vec{A}$.

Choose Landau gauge:

$$\vec{A} = x B \hat{y}$$

$$H = \frac{1}{2m}\left(-i\hbar\vec{\nabla} + \frac{e}{c}\vec{A}\right)^2 = \frac{1}{2m}\left(-i\hbar\vec{\nabla} + \frac{eB}{c}x\hat{y}\right)^2 =$$

$$= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial y} + \frac{eB}{c}x\right)^2$$

I dropped the star superscript for the effective mass.

Need to solve $H\Psi = E\Psi$.

Look for solution in the form:

$$\Psi(x,y) = e^{iky}\varphi(x)$$

We choose this form because H is translationally invariant in the y-direction $\Rightarrow$ solution should be a plane wave, but not in the x-direction.

Plug this ansatz into the Schrodinger equation.

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + \frac{1}{2m}\left(\hbar k + \frac{eB}{c}x\right)^2\varphi = E\varphi$$

Rewrite in terms of cyclotron frequency $\omega_c = \frac{eB}{mc}$ and the magnetic length $l = \sqrt{\frac{\hbar c}{eB}}$:

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} + \frac{m\omega_c^2}{2}\left(x+k\ell^2\right)^2\varphi = E\varphi$$

This is the Hamiltonian of a harmonic oscillator of frequency ω_c, centered at $x=-k\ell^2$.

The eigenstates and the energy spectrum have the following form:

$$E_{nk} = \hbar\omega_c\left(n+\frac{1}{2}\right), \quad n = 0, 1, 2, \ldots$$

$$\varphi_{nk}(x) = \frac{1}{\sqrt{\ell\sqrt{\pi}\,2^n n!}}\, e^{-\frac{1}{2\ell^2}(x+k\ell^2)^2} H_n\left(\frac{x+k\ell^2}{\ell}\right)$$

H_n is a Hermite polynomial.

Thus the spectrum of electrons is drastically changed: instead of $\mathcal{E}_k = \frac{\hbar^2k^2}{2m}$ we have a set of discrete levels. These are called Landau levels.

Note that E_{nk} are independent of $k \Rightarrow$ each Landau level contains an infinite (in the thermodynamic limit) number of degenerate states, corresponding to different values of k.
Let's find the allowed values of k.

Assume periodic boundary conditions in the y-direction:

$$\psi(x, y+L_y) = \psi(x, y)$$

As usual, this means that:

$$K = \frac{2\pi}{L_y} n \quad , n = 0, \pm 1, \pm 2, \ldots$$

But note that k also ~~[illegible]~~ determines the center-of-mass position of $\varphi_{nk}(x)$ in ~~[illegible]~~ the x-direction.

This means that:

$$-L_x \le -k\ell^2 \le 0 \quad \text{or} \quad 0 \le k\ell^2 \le L_x.$$

Thus k can run from 0 to $\frac{L_x}{\ell^2}$.

The total number of states in each Landau level:

$$N = \frac{L_x/\ell^2}{\frac{2\pi}{L_y}} = \frac{L_x L_y}{2\pi \ell^2}$$

Thus $2\pi\ell^2$ has the meaning of area per one state.

This can be written as:

$$N = \frac{L_x L_y}{2\pi \ell^2} = \frac{L_x L_y e B}{2\pi \hbar c} = \frac{\Phi}{\Phi_0}$$

$\Phi = L_x L_y B$ - total magnetic flux ~~[illegible]~~ through the sample.

$$\Phi_0 = \frac{2\pi \hbar c}{e} = \frac{hc}{e} \text{ - magnetic flux quantum.}$$

$$\Phi_0 = 2.07 \times 10^{-7} \text{ gauss} \cdot \text{cm}^2.$$