

Lecture 1

What is Solid State Physics?

Quantum mechanics: properties of electrons and individual atoms

Solid State Physics: what happens when 10^{23} atoms are brought together?

Typically (low enough T , high enough P) macroscopic collections of atoms form solids.

Solids are distinguished by "rigidity":

solids are rigid (e.g. you can stand on ice),
liquids are not (you drown when the ice melts).

Solids are most frequently crystalline: atoms form regular arrays or crystal lattices.

Why large numbers of atoms brought together form crystalline solids? This can not be demonstrated ~~by~~ by any calculation and should be regarded as experimental fact.

In this class we will focus on electronic properties of solids.

We will for the most part assume that crystal lattice forms a static background

potential, in which interacting electrons move.

Justification: electrons are much lighter and move much faster than the ionic cores.

Just like large collections of interacting atoms exhibit emergent organizational phenomena, e.g.

formation of ~~organized structures~~ rigid crystalline solids, large numbers of interacting electrons moving in the crystalline background organize themselves in different macroscopic phases.

Examples:

1. Metals - conduct electricity at $T=0$.
2. Insulators - don't conduct at $T=0$.
3. Semiconductors - technologically important subclass of insulators.
4. Ferromagnets - have spontaneous macroscopic magnetic moment at low enough T .
5. Superconductors - zero resistance to current flow, expel magnetic field.
6. Quantum Hall systems - precisely quantized Hall resistance, incompressible, have fractionally charged excitations.

Our goal - describe these phases and get some idea for how they arise.

Emphasize: these are ~~old~~ emergent properties, characterizing ~~large~~ macroscopic collections of atoms.

Try to ~~also~~ set up the problem of calculating electronic properties of a solid formally.

Recall single-particle quantum mechanics.

Single electron, moving in some potential, obeys

Schrodinger equation:

$$\left[-\frac{\hbar^2 \nabla^2}{2m} + V(\vec{r}) \right] \Psi(\vec{r}) = E \Psi(\vec{r})$$

$\Psi(\vec{r})$ - wavefunction, $|\Psi(\vec{r})|^2$ - probability to find electron at point \vec{r} .

This equation can always be solved (sometimes numerically) - quantum mechanics is simple in this sense.

What about electrons in a solid?

Suppose we have a crystalline sample with $N \sim 10^{23}$ ion cores forming the crystal lattice. Assume they form a static potential.

Each atom of the crystal lattice has valence electrons i.e. electrons in the outer shell of atoms — these electrons will be able to move through the crystal and these are the electrons we are interested in. The others will be bound to individual atoms and form part of the ionic core.

Atom of valence Z has Z valence electrons \Rightarrow

\Rightarrow The total number of such electrons is ZN .

Let me assume for simplicity $Z=1$ (alkali metals).

The Hamiltonian for such a system of electrons can be written as:

$$H = \sum_{i=1}^N -\frac{\hbar^2 \nabla_i^2}{2m} + \frac{1}{2} \sum_{i,j=1}^N \frac{e^2}{|\vec{r}_i - \vec{r}_j|} + \sum_{i=1}^N V(\vec{r}_i)$$

\vec{r}_i — coordinate of i 's electron.

∇_i means that the operator acts on i 's coordinate

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 $V(\vec{r})$ is the periodic potential due to ion cores.

This Hamiltonian acts on a wavefunction which is a function of coordinates ~~and spins~~ of all the electrons:

$$\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N)$$

~~We~~ have to solve $H\Psi = E\Psi$.

This is a many-body quantum mechanics problem.

Unlike ~~the~~ single particle Schrödinger equation which can always be solved, many-body Schrödinger equation ~~for~~ 10^{23} particles can never be solved!

Starting ~~from~~ SE and trying to solve it is a wrong approach in principle - ~~properties~~

physical properties that we are after are emergent, i.e. not reducible to those of individual atoms and electrons.

Need a completely different approach. Solid state physics does not reduce to ~~complicated~~ solving a complicated SE.

Introduce a few numbers and parameters that will be useful.

Units: I will use CGS units throughout.

Important parameters:

Electron density $n = \frac{N}{V} \sim 10^{22} \text{ cm}^{-3}$

Useful related parameter: volume per electron.

$$\frac{V}{N} = \frac{1}{n} \equiv \frac{4\pi r_s^3}{3}$$

$$r_s = \left(\frac{3}{4\pi n} \right)^{1/3} - \text{radius of a sphere, containing one electron.}$$

$$a_0 \equiv \frac{\hbar^2}{me^2} = 0.5 \cdot 10^{-8} \text{ cm} = 0.5 \text{ \AA} - \text{Bohr radius - typical microscopic length scale.}$$

$$\frac{r_s}{a_0} \sim 1-10 \text{ in most metallic solids.}$$

Let's estimate relative importance of kinetic and interaction energies for electrons in a solid.

$$E_{\text{kin}} \sim \frac{\hbar^2}{m r_s^2} - \text{minimum kinetic energy electron}$$

acquires due to uncertainty principle if you try to localize it in a box of size r_s .

$E_{int} \sim \frac{e^2}{r_s}$ - typical Coulomb interaction energy
if the average distance between electrons is r_s .

$$\frac{E_{int}}{E_{kin}} = \frac{e^2}{r_s} \cdot \frac{m r_s^2}{\hbar^2} = \frac{r_s}{\frac{\hbar^2}{m e^2}} = \frac{r_s}{a_0}$$

If $\frac{r_s}{a_0} \ll 1$ - we could neglect interaction between electrons, ~~the~~ problem would become simple.

But typical $\frac{r_s}{a_0} \sim 1-10$ - can't ignore interactions.

Other important numbers:

$e = 4.8 \cdot 10^{-10}$ esu - electron charge.

$m = 9.1 \cdot 10^{-28}$ g.

Typical energy scale:

$$E \sim \frac{\hbar^2}{m r_s^2} \sim \frac{10^{-54} \text{ erg}^2 \cdot \text{s}^2}{10^{-27} \text{ g} \cdot 10^{-15} \text{ cm}} \sim 10^{-12} \text{ erg}$$

Natural energy unit is electronvolt $1 \text{ eV} = 1.6 \cdot 10^{-12} \text{ erg}$.