Homework 4
Due Wednesday, October 28 in class

1. Assume electrons are moving on a two-dimensional square lattice with lattice constant a. The tight-binding band dispersion is

$$\epsilon(\mathbf{k}) = -2\gamma[\cos(k_x a) + \cos(k_y a)],$$

where γ is the overlap integral.

   (a) Draw Fermi surfaces in three cases: almost empty band, exactly half-filled band, and nearly full band. What is the electron density corresponding to half-filled band?

   (b) Calculate effective masses near the bottom and near the top of the band.

   (c) Assume a constant magnetic field is applied in the z-direction, i.e. perpendicular to the plane in which electrons move. Write down quasiclassical equations of motion. Using these, prove that the energy is a constant of motion (this can be proven generally, without using the explicit form of the band dispersion). This means that electron trajectories in momentum space trace out constant energy contours. Find trajectories numerically (use Maple) in three cases: electrons with energy near the bottom of the band, in the middle of the band and near the top of the band. Compare with your Fermi surface sketches.

   (d) Solve quasiclassical equations of motion analytically near the bottom of the band, using effective mass approximation for the band dispersion.

2. For the square lattice tight-binding band from Problem 1 calculate the conductivity for half-filled band. Use the general expression for the conductivity we obtained in class:

$$\sigma = \frac{e^2\tau}{4\pi^2\hbar}\int d\ell_F\ v_{\mathbf{k}},$$

and the Fermi surface for half-filled band you found in Problem 1. Compare the result with what you would get if you used effective mass approximation near the bottom of the band for the dispersion.