Homework 6
Due Monday, November 16 in class

1. Consider electrons in a two-dimensional sample with dimensions L_x and L_y. Assume there are $N = nL_xL_y$ electrons and assume that the sample is so clean that the impurity potential can be neglected.

   (a) Calculate and plot the magnetic field dependence of the chemical potential μ for temperatures $k_BT \ll \hbar\omega_c$, where ω_c is the cyclotron frequency.

   (b) Real samples are of course never perfectly two-dimensional. Assume the sample thickness is $d = 100\mathring{A}$ and let the electron density be $n = 10^{18}\text{cm}^{-3}$. What is the typical magnetic field required to observe the integer quantum Hall effect? What temperatures are low enough for this field?

2. Show that the free energy of *classical* particles with no internal magnetic moment is always independent of magnetic field. This means that all phenomena, involving the dependence of thermodynamic properties of the electron gas on magnetic field, are intrinsically quantum mechanical. *Hint: write down the partition function Z for a collection of N classical particles. Let the particles interact with each other via an arbitrary potential U, which depends only on the positions of the interacting particles. Show that the magnetic field dependence can be eliminated from the partition function by a variable change.*

3. This problem illustrates why disorder is crucial for the existence of the quantum Hall effect. Consider an infinite disorder-free two-dimensional sample with electron density n in a constant uniform perpendicular magnetic field $\mathbf{B} = B\hat{z}$. Such a sample is clearly translationally invariant and there is no preferred frame of reference. Assume there is no electric field applied in the lab frame.

   (a) Transform to a reference frame moving with velocity $-\mathbf{v}$ with respect to the lab frame. What is the current density, carried by electrons in this frame?

   (b) To lowest order in v/c, what are the electric and magnetic fields in the moving frame?

(c) From the above expressions for **E** and **B** find the resistivity tensor. Discuss the meaning of the result. Why does the presence of disorder invalidate the above derivation of the resistivity tensor?