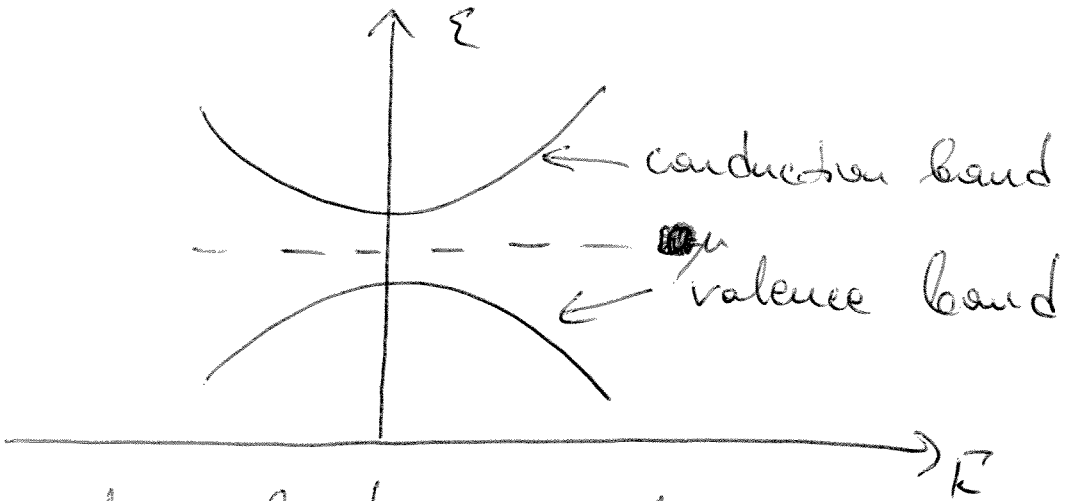
Lecture 19

Semiconductors

Semiconductors are insulators with a small bandgap ~ 0.1 – 2 eV.

Cartoon bandstructure of a semiconductor:



Valence band - completely filled at T=0,
conduction band - unfilled at T=0.

Semiconductors are important technologically since it is easy to tune the density of carriers (electrons in conduction band and/or holes in the valence band) externally, thus tuning their transport properties.

The tuning can be accomplished by applying external voltage, doping, etc.

Since undoped semiconductor at T=0 has no holes in the valence band and no electrons in the conduction band, we can expect the effective mass approximation to be very good, since we will be dealing with valence band lightly filled with holes and conduction band lightly filled with electrons.

We will approximate the dispersion as:

$$\varepsilon_{c\vec{k}} = \varepsilon_c + \frac{\hbar^2 k^2}{2m_c^*} \quad - \text{conduction band}$$

$$\varepsilon_{v\vec{k}} = \varepsilon_v - \frac{\hbar^2 k^2}{2m_v^*} \quad - \text{valence band}$$

ε_c - bottom of the conduction band.

ε_v - bottom of the valence band.

$\varepsilon_c - \varepsilon_v = \varepsilon_{gap}$

The most important property of a semiconductor is the density of electrons in the conduction band and holes in the valence band at a given temperature.

$$n(T) = \int_{\varepsilon_c}^{\infty} d\varepsilon \, g_c(\varepsilon) \, n_F(\varepsilon - \mu) =$$

$$= \int_{\varepsilon_c}^{\infty} d\varepsilon \, g_c(\varepsilon) \frac{1}{e^{\frac{\varepsilon-\mu}{k_B T}} + 1}$$ - density of electrons in conduction band.

$$p(T) = \int_{-\infty}^{\varepsilon_v} d\varepsilon\, g_v(\varepsilon)\left(1 - n_F(\varepsilon-\mu)\right) =$$

$$= \int_{-\infty}^{\varepsilon_v} d\varepsilon\, g_v(\varepsilon)\; n_F(\mu-\varepsilon) =$$

$$= \int_{-\infty}^{\varepsilon_v} d\varepsilon\, g_v(\varepsilon)\, \frac{1}{e^{\frac{\mu-\varepsilon}{k_B T}} + 1}$$

$p(T)$ - density of holes in the valence band.

Assume the chemical potential is far from either the bottom of the conduction band or top of the valence band:

$$\varepsilon_c - \mu \gg k_B T$$

$$\mu - \varepsilon_v \gg k_B T$$

These conditions define a "nondegenerate semiconductor".

What this means physically is that the densities of electrons and holes are very small $\Rightarrow$ the fact that they are fermions obeying Fermi-Dirac statistics is unimportant.

$$\frac{1}{e^{\frac{\varepsilon-\mu}{k_B T}} + 1} \approx e^{-\frac{\varepsilon-\mu}{k_B T}}, \quad \varepsilon > \varepsilon_c$$

$$\frac{1}{e^{\frac{\mu-\varepsilon}{k_B T}}+1} \approx e^{-\frac{\mu-\varepsilon}{k_B T}}, \quad \varepsilon < \varepsilon_v$$

Then we have:

$$n(T) = \int_{\varepsilon_c}^{\infty} d\varepsilon \; g_c(\varepsilon) e^{-\frac{\varepsilon-\mu}{k_B T}} =$$

$$= \int_{\varepsilon_c}^{\infty} d\varepsilon \; g_c(\varepsilon) e^{-\frac{\varepsilon-\varepsilon_c}{k_B T}} e^{-\frac{\varepsilon_c-\mu}{k_B T}} =$$

$$= N_c(T) e^{-\frac{\varepsilon_c-\mu}{k_B T}}$$

$$N_c(T) = \int_{\varepsilon_c}^{\infty} d\varepsilon \; g_c(\varepsilon) e^{-\frac{\varepsilon-\varepsilon_c}{k_B T}}$$

If the effective mass approximation is valid, the density of states is given by:

$$g_c(\varepsilon) = \frac{m_c^*}{\pi^2 \hbar^2} \sqrt{\frac{2m_c^*}{\hbar^2}(\varepsilon-\varepsilon_c)}$$

$$g_v(\varepsilon) = \frac{m_v^*}{\pi^2 \hbar^2} \sqrt{\frac{2m_v^*}{\hbar^2}(\varepsilon_v-\varepsilon)}$$

Evaluating the integral in the expression for $N_c(T)$, we obtain:

$$N_c(T) = 2\left(\frac{m_c^* k_B T}{2\pi\hbar^2}\right)^{3/2}$$

Analogously for the holes:

$$p(T) = P_v(T)\, e^{-\frac{\mu - E_v}{k_B T}}$$

$$P_v(T) = 2\left(\frac{m_v^* k_B T}{2\pi\hbar^2}\right)^{3/2}$$

Consider the product np:

$$np = N_c P_v e^{-\frac{E_c - E_v}{k_B T}} = N_c P_v e^{-\frac{E_{gap}}{k_B T}}$$

The important thing to notice here is that this is independent of the chemical potential.

Consider now the case of an <u>intrinsic semiconductor</u> = undoped semiconductor.

In this case the electron and hole densities must be equal:

$$n = p = n_i = (np)^{1/2}$$

$$n_i = (N_c P_v)^{1/2} e^{-\frac{E_{gap}}{2k_B T}}$$

$$N_c(T) \approx 2.5\left(\frac{m_c^*}{m}\right)^{3/2}\left(\frac{T}{300k}\right)^{3/2} \times 10^{19}\ cm^{-3}$$

This shows that 10^{-19} cm^{-3} is about as high as the carrier density in an intrinsic semiconductor can go.

Find μ in an intrinsic semiconductor.

Consider $p = n_i$

$$p_v e^{-\frac{\mu - E_v}{k_B T}} = (N_c p_v)^{1/2} e^{-\frac{E_{gap}}{2k_B T}}$$

Take a logarithm of both sides:

$$\ln p_v - \frac{\mu - E_v}{k_B T} = \frac{1}{2}\ln N_c + \frac{1}{2}\ln p_v - \frac{E_{gap}}{2k_B T}$$

Solving this for μ we obtain:

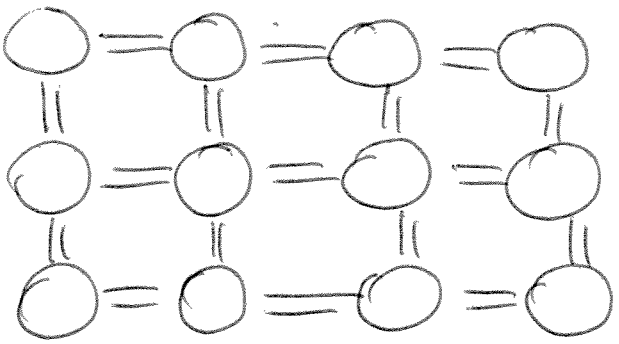
$$\mu = E_v + \frac{1}{2}E_{gap} + \frac{3}{4}k_B T \ln\frac{m_v}{m_c}$$

Thus at low T μ is close to midgap.

Now consider extrinsic semiconductors = doped semiconductors.

The idea of doping is very simple — take a host semiconductor atom and replace it with an atom of higher (donors) or lower (acceptors) valence.

Roughly — donors contribute electrons to the conduction band while acceptors contribute holes to the valence band



○ = Ge, 4 valence electrons, ● each participating in covalent bonding with neighboring atoms.

Now we replace one Ge atom by As.

As has 5 valence electrons. 4 of them will be involved in bonding with the neighboring atoms, leaving 1 electron "free". This electron will leave the As atom, which will be positively charged as a result.

Typical host - dopant pairs:

Donors:

Si ↔ P, As

Ga ↔ Si, Ge

Acceptors

Si ↔ B, Al

Ga ↔ Mg, Be

. Electron, provided by a donor impurity, is of course not completely free. Electron has charge $-e$ and donor atom $+e$ ⇒ they will interact via Coulomb interaction.

Simple model for electron–ionized donor interaction:

electron of charge $-e$ and mass m^* moving in the presence of a fixed positive charge $+e$, in a medium with dielectric constant ε.

Two things are crucial for why doping works:

$m^* < m$ and $\varepsilon \gg 1$.

$\varepsilon \gg 1$ is due to the fact that semiconductors are insulators with small gaps ⇒ not too far from metals, for which the static ε is infinite.

This problem is very like hydrogen atom but in a medium and with a different electron mass. In the hydrogen atom results we need to replace e^2 by $\frac{e^2}{\varepsilon}$ and m by m^*.

Bohr radius: $a_0 = \frac{\hbar^2}{m e^2}$ is replaced by:

$$r_0 = \frac{\hbar^2 \varepsilon}{m^* e^2} = \frac{m}{m^*} \varepsilon \, a_0 \gg a_0$$

. Ground state energy = Binding energy:

$$E_0 = \frac{me^4}{2\hbar^2} = 13.6\ eV$$

This is replaced by:

$$E_B = \frac{m^* e^4}{2\hbar^2 \varepsilon^2} = \frac{m^*}{m}\frac{1}{\varepsilon^2} E = \frac{m^*}{m}\frac{1}{\varepsilon^2} \times 13.6\ eV$$

Typical values $r_0 \sim 100\ \text{Å}$, $E_B \sim 0.01\ eV$.

This is why doping is possible and useful – much easier to excite electrons from donor levels than across the gap.