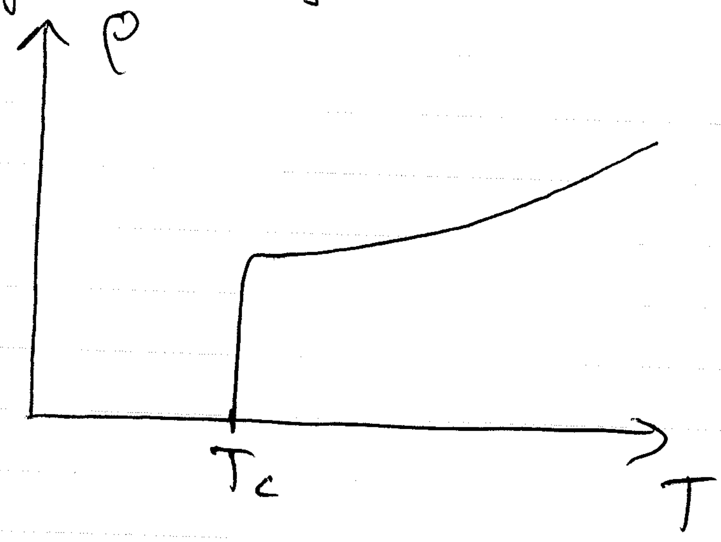
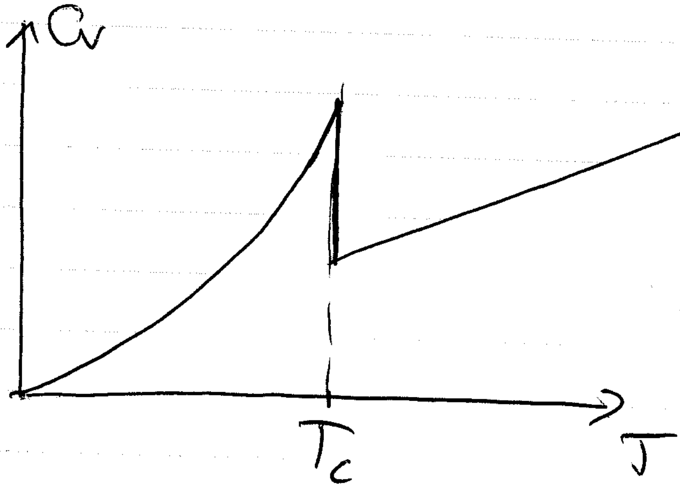
lecture 32

Superconductivity

Most metallic elements (except ferromagnets) and even some nonmetals undergo a transition into a superconducting state as the temperature is lowered.



- electrical resistivity vanishes below T_c.

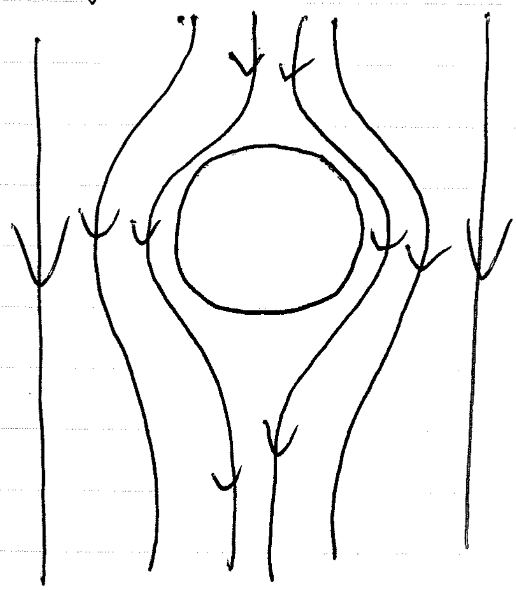


Specific heat has a jump at T_c.
T-dependence changes from $C_v \sim T$ to $C_v \sim e^{-\frac{\Delta}{k_B T}}$.

Superconducting state is a new state of matter – it can not be thought of as a "metallic state with zero resistance".

The effect that clearly demonstrates that superconductor is not just a metal with zero resistance is the Meissner effect.
The effect has to do with the behavior of a superconductor in an external magnetic field.

First assume superconductor is simply an ideal conductor, i.e. just a metal with zero resistance. Suppose we cooled the sample down below T_c where $\rho = 0$. Then we apply external magnetic field.



– The field inside a conductor with $\rho = 0$ is zero.

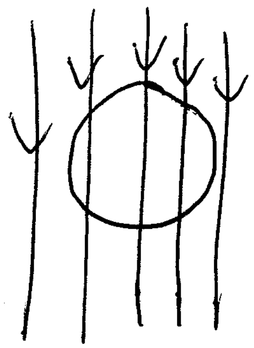
$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$\vec{E} = \rho \vec{j} \Rightarrow \vec{E} = 0$ inside an ideal conductor $\Rightarrow \dfrac{\partial \vec{B}}{\partial t} = 0$.

B applied at $T < T_c$

Since $\vec{B}$ was 0 before applying the field, it will remain zero.

Now imagine a different experiment: let's apply the field at $T > T_c$ and cool the sample down afterwards.



— B applied at $T > T_c$

In this case we again have $\frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow \vec{B} = \text{const}$.

The field inside an ideal conductor will remain the same as before cooling below T_c.

Superconductor behaves differently:

magnetic field inside a superconductor below T_c is always zero. — Meissner effect.

Ideal conductor:

$\rho = 0, \quad B = \text{const}$

Superconductor:

$\rho = 0, \ B = 0$

This clearly demonstrates that superconductor is a distinct state of matter, not just a metal in a zero resistivity limit.

Temperature scales:

In superconductors based on elemental metals (Al, Sn, Hg)
$T_c \sim 1-10\,K$ - requires liquid He temperatures to be observed.

High-T_c-superconductors: $La_{2-x}Sr_xCuO_4$, $T_c = 34K$

$YBa_2Cu_3O_{6+x}$, $T_c = 92K$

$Bi_2Sr_2Ca_1Cu_2O_{8+x}$ $T_c = 95K$

$HgBa_2Ca_2Cu_3O_8$ $T_c = 133K$.

Microscopic origin of superconductivity: pairing of electrons.

In metallic superconductors this is due to exchange of phonons: electron moving through the crystal lattice, polarizes it locally, which attracts another electron.

In cuprate high-T_c superconductors pairing is thought to be due to the antiferromagnetic spin-spin interaction of the parent (undoped) Mott insulator.

The most important qualitative properties of superconductors can be described using semiphenomenological Landau-Ginzburg theory.

Pairing typically occurs between electrons of opposite spin.
Let $\Phi^+(\vec{r}) = \Psi_\uparrow^+(\vec{r})\Psi_\downarrow^+(\vec{r})$, where $\Psi_\sigma^+(\vec{r})$ are electron field operators.

A pair of fermions is a boson $\Rightarrow$ $\Phi^+(\vec{r})$ creates a bosonic particle.
Rewrite $\Phi^+(\vec{r})$ in momentum representation:

$$\Phi^+(\vec{r}) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} b_k^+ e^{-i\vec{k}\cdot\vec{r}}$$

Here b_k^+ creates a bosonic particle (electron pair) with momentum $\vec{k}$.

$$[b_k, b_{k'}^+] = \delta_{k,k'}$$

While electrons obey Pauli principle — no two electrons can occupy the same state, any number of bosons can occupy the same quantum state.
It follows that at low T bosons tend to congregate in the lowest energy states, typically the states with small $\vec{k}$.

$$b_k^+ b_k |\ldots n_k \ldots\rangle = n_k |\ldots n_k \ldots\rangle$$

$$b_k b_k^+ |\ldots n_k \ldots\rangle = (1 + b_k^+ b_k)|\ldots n_k \ldots\rangle =$$

$$= (n_k + 1)|\ldots n_k \ldots\rangle$$

When n_k is very large, we can neglect

The difference between the action of $b_k^+ b_k$ and $b_k b_k^+$.

This means that in the limit of large occupation numbers b_k^+, b_k ~~associated to the~~ cease to be quantum operators and become simple commuting complex numbers.

$$\varphi^+(\vec{r}) = \frac{1}{\sqrt{V}} \sum_k b_k^+ e^{-i\vec{k}\cdot\vec{r}}$$

With the sum over $\vec{k}$ restricted to only small momenta with macroscopic occupation $\varphi(\vec{r})$ also becomes just a complex function.
Its physical meaning is then simply a wavefunction, describing the condensate of electron pairs.
The condensation of electron pairs into a state described by a single wavefunction $\varphi(\vec{r})$ is the physical origin of superconductivity.

Just as the free energy of a ferromagnet can be written as an expansion in powers of the macroscopic magnetization near the transition temperature, free energy of a superconductor can be written as an expansion in powers of the consensate wavefunction $\varphi(\vec{r})$.

For a homogeneous superconductor in the absence of magnetic field, we can write:

$$F_s = F_n + a|\varphi|^2 + \frac{b}{2}|\varphi|^4$$

Minimizing this with respect to $|\varphi|^2$, we obtain:

$$\frac{dF_s}{d|\varphi|^2} = a + b|\varphi|^2 = 0.$$

$$|\varphi|^2 = -\frac{a}{b}$$

Assuming $a \sim T - T_c$, there is a nonzero solution for $|\varphi|^2$ when $T<T_c$ and there is no solution when $T>T_c$.

For $T<T_c$, $|\varphi| = \sqrt{-\frac{a}{b}} = \sqrt{\frac{|a|}{b}}$

$|\varphi| \sim \sqrt{T_c - T}$ near T_c, just like magnetization in a ferromagnet in MFT (MFT in ordinary superconductors turns out to be nearly exact, unlike in ferromagnets).

To describe Meissner and other effects, we want to generalize the above free energy expansion to inhomogeneous case in the presence of magnetic field.

Recall that the kinetic energy part of the Hamiltonian in second quantized notation has the form:

$$H = \int d\vec{r}\, \varphi^*(\vec{r}) \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)^2 \varphi(\vec{r}) =$$

$$= \frac{1}{2m^*}\int d\vec{r} \left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\varphi(\vec{r})\right|^2$$

Here I've used the fact that momentum is a Hermitian operator.

$m^* = 2m$, $e^* = 2e$.

Thus we can write the expansion in the general case as:

$$F_s = F_n + \int d\vec{r} \left[\frac{1}{2m^*} \left| \left(-i\hbar \vec{\nabla} + \frac{e^*}{c} \vec{A} \right) \Phi(\vec{r}) \right|^2 + \right.$$

$$\left. + a |\Phi|^2 + \frac{b}{2} |\Phi|^4 + \frac{\vec{B}^2}{8\pi} - \frac{\vec{B} \cdot \vec{H}}{4\pi} \right]$$

Here $\vec{H}$ is the external field.