Homework 5
Due Friday, November 6 in class

1. In class we calculated the DC conductivity of a metal (response to a time-independent electric field) by solving a static Boltzmann equation. If the applied electric field is not static, time-dependent Boltzmann equation needs to be solved.

   (a) Assume the time dependence of the electric field is of the form $\mathbf{E}(t) = \mathbf{E}e^{-i\omega t}$. Solve the time-dependent linearized Boltzmann equation and find the frequency-dependent conductivity $\sigma(\omega)$. *Hint: assume that the time dependence of the distribution function is of the form* $f_1 \sim e^{-i\omega t}$. Plot $\mathrm{Re}[\sigma(\omega)]$ versus ω. The characteristic shape of this dependence is often referred to as the *Drude peak*.

   (b) Now assume that electric field is applied at time $t = 0$ and held constant afterwards. Find the current density $\mathbf{j}(t)$ for all $t > 0$.

   You can assume that the effective mass approximation is valid in both cases.

2. Assume conduction band dispersion in a semiconductor has the form

$$\epsilon_c(\mathbf{k}) = \epsilon_c + \frac{\hbar^2 k_x^2}{2m_{cx}} + \frac{\hbar^2 k_y^2}{2m_{cy}} + \frac{\hbar^2 k_z^2}{2m_{cz}}.$$

   Prove that the density of states is given by:

$$g(\epsilon) = \frac{m_c^{3/2}}{\pi^2 \hbar^3} \sqrt{2|\epsilon - \epsilon_c|},$$

   where $m_c = (m_{cx} m_{cy} m_{cz})^{1/3}$.