Lecture 26

Magnetism

Simplest and best known magnetic phenomenon – ferromagnetism

Ferromagnetism – the appearance of spontaneous macroscopic magnetic moment below certain temperature in some metals.

All ferromagnetic metals have one thing in common; incompletely filled d-shells.

For example, iron:

$3d^6 4S^2$

d-shell can have a maximum of 10 electrons ⇒
⇒ d-shell in iron is not completely filled.

Any completely filled shell has zero total spin ⇒ zero magnetic moment.

~~[illegible]~~

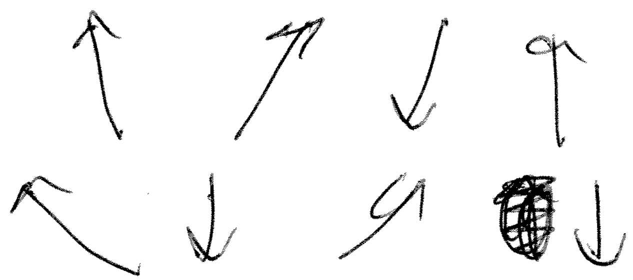
Incompletely filled shell: ~~[illegible]~~ shell is filled in such a way as to maximize the total spin (Hund's rule).
Thus any incompletely filled shell has nonzero spin ⇒ nonzero magnetic moment.

But magnetism is not observed in metals with incompletely filled s or p-shells, only d or f.

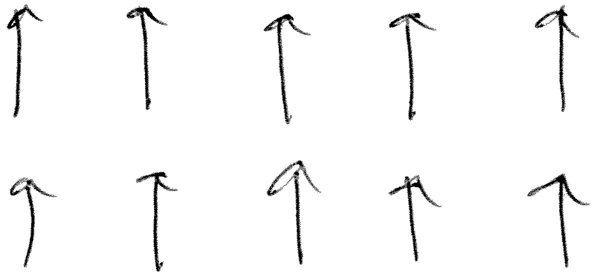
The difference is that d-electrons are significantly stronger localized than s or p-electrons.

Thus magnetism is associated with partially localised magnetic moments. However they are only partially localised: d-electrons are delocalized and participate in conduction in ferromagnetic metals: this is one of the things that makes theory of ferromagnetism very complicated.

Even if there are microscopic magnetic moments, if they don't interact there is no ferromagnetism, since each moment fluctuates randomly and independently, adding up to zero macroscopic moment.



To produce macroscopic magnetic moment, these atomic moments need to align:



Such an alignment can only result from interaction between the moments.
The most basic and fundamental question in magnetism: what is the source of this interaction.

Naively magnetic moments always interact directly through the magnetic dipole interaction:

$$U_{dip} = \frac{1}{r^3}\left[\vec{\mu}_1 \cdot \vec{\mu}_2 - 3\left(\vec{\mu}_1 \cdot \hat{r}\right)\left(\vec{\mu}_2 \cdot \hat{r}\right)\right]$$

Maybe this is already enough to explain magnetism!

let's estimate the magnitude of U_{dip}

Magnetic moment of the electron is $\vec{\mu} = -g\mu_B \vec{S}$

$\vec{S}$ - spin of the electron.

g - Lande factor ≈ 2.

$$\mu_B = \frac{e\hbar}{2mc} \text{ - Bohr magneton.}$$

Thus typical atomic magnetic moments have magnitude:

$$\mu \approx g\mu_B = \frac{e\hbar}{mc}$$

$$U_{dip} \sim \frac{(g\mu_B)^2}{r^3} = \left(\frac{e\hbar}{mc}\right)^2 \frac{1}{r^3} =$$

$$= \left(\frac{e^2}{\hbar c}\right)^2 \frac{e^2}{a_0}\left(\frac{a_0}{r}\right)^3 = \alpha^2 \frac{e^2}{a_0}\left(\frac{a_0}{r}\right)^3$$

$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137} \text{ - fine structure constant.}$$

$$a_0 = \frac{\hbar^2}{me^2} \text{ - Bohr radius}$$

$$a_0 \approx 0.5 \text{ Å}$$

Typical interatomic separation in metals:

$$r \simeq 1-2 \text{ Å}.$$

Then $U_{dip} \sim 10^{-4}$ eV.

This corresponds to temperature of about 1K.

However, the critical temperature of Fe is 1043K, thousand times larger energy scale.
Thus magnetic dipole interaction can't explain ferromagnetism and plays no role in it.

It turns out that what actually leads to sufficiently strong interaction between the atomic magnetic moments is the combination of Pauli principle and Coulomb interaction.

$$V_{coulomb} \sim \frac{e^2}{r} = \frac{e^2}{a_0}\frac{a_0}{r}$$ - this is more than 137^2 times larger than U_{dip}.

How Coulomb interactions and Pauli principle lead to spin alignment is best illustrated on the simplest example — two interacting electrons.