University of Waterloo
Department of Electrical and Computer Engineering

ECE 413 – Digital Signal Processing
Final Exam, Spring 2009

August 13, 2009, 9:00-11:30 AM

Instructor: Dr. Oleg Michailovich

Student's name: _____

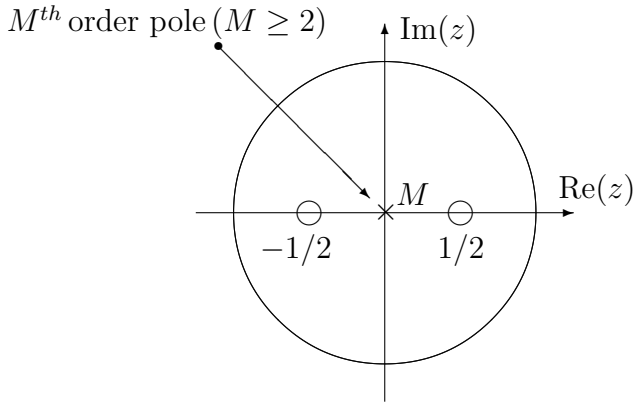
Student's ID #: _____

Instructions:

- This exam has **4** pages.
- **No books and lecture notes are allowed on the exam**. Please, turn off your cell phones, PDAs, etc., and place your bags, backpacks, books, and notes under the table or at the front of the room.
- Please, place your **WATCARD** on the table, and fill out the exam attendance sheet when provided by the proctor after the exam starts.
- Question marks are listed by the question.
- Please, do not separate the pages, and indicate your Student ID at the top of every page.
- Be neat. Poor presentation will be penalized.
- **No questions will be answered during the exam**. If there is an ambiguity, state your assumptions and proceed.
- **No student can leave the exam room in the first 45 minutes or the last 10 minutes**.
- If you finish before the end of the exam and wish to leave, remain seated and raise your hand. A proctor will pick up the exam from you, at which point you may leave.
- When the proctors announce the end of the exam, put down your pens/pencils, close your exam booklet, and remain seated in silence. The proctors will collect the exams, count them, and then announce you may leave.

Problem №1 (25%)

Consider the following zero-pole plot of $H(z)$:

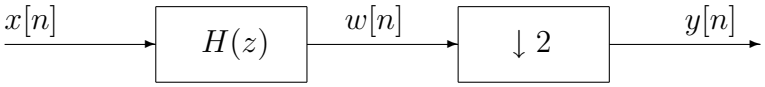


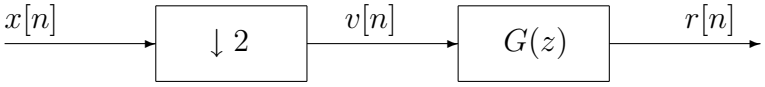
Given that $H(1) = 3/4$ and the the region of convergence (ROC) of $H(z)$ is $|z| > 0$:

a) Determine $H(z)$.

b) Determine wether $H(z)$ is: stable, causal, IIR, FIR, minimum-phase, and/or possessing a generalized linear phase (GLP).

c) Consider the following system





Find $G(z)$ such that $y[n] = r[n]$.

Problem №2 (25%)

Consider the following two sequences:

$$x[n] = \{-1, -3, -1, 5\}, \qquad w[n] = \{0, 7, 0, 5\}.$$

a) Determine if a sequence that satisfies $x \circledast y = w$ can be found. If so, find $y[n]$. If not, prove it does not exist.

b) Given that $W[k] = \text{DFT}_4\{w[n]\} = \text{Im}\{G[k]\}$, find $g[3] - g[1]$.

c) Find $q[n]$ whose DFT is given by: $Q[k] = W[(k-2) \bmod 4]\, W_4^{2k}$.

Problem №3 (15%)

A DFT engineer would like to use windowing and zero padding to analyze the DFT of a sampled signal.

a) Given that the signal was sampled using a sampling frequency of 128 Hz, and that the engineer intends to use a rectangular window and apply the DFT of length $N = 64$ to the signal, what is the minimal distance (in Hz) between two distinguishable frequency components of the signal?

b) Which of the following two is the proper procedure:

- Zero-pad, multiply by a window, apply FFT;
- Multiply by a window, zero-pad, apply DFT.

Problem №4 (25%)

In STFT analysis, the DTFT is applied to windowed versions of a sequence $x[n]$, namely

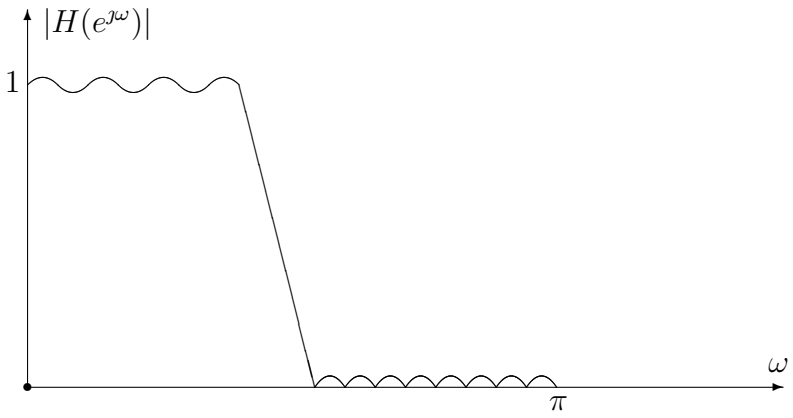
$$X[n, \lambda) = \text{DTFT}\{x[m + n]\, w[m]\} = \sum_{m=-\infty}^{\infty} x[n + m]w[m]e^{-\jmath\lambda m}.$$

Alternatively, one can apply the DTFT to the auto-correlation sequence $a[n]$ of the windowed signal $x[m + n]\, w[m]$ which is defined as:

$$a[m, n] = \sum_{l=-\infty}^{\infty} x[n + l]\, w[l]\, x[n + m + l]\, w[m + l].$$

Find the DTFT $A[n, \lambda)$ of $a[m, n]$ in terms of $X[n, \lambda)$. (Assume real sequence and window.)

Problem №5 (10%)



Considering the above response of an FIR generalized linear phase (GLP) filter, which of the following statements is correct:

a) The filter is Type I;

b) The filter is Type II;

c) The filter is Type III;

d) The filter type cannot be determined from the graph.

Explain your answer.