Home Project №4 - Solution

Due on July 10, 2008

Exercise 1

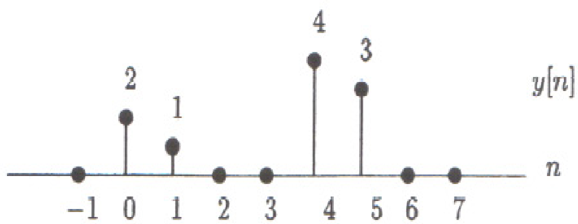
(a) When multiplying the DFT of a sequence by a complex exponential, the time-domain signal undergoes a circular shift.

For this case,

$$Y[k] = W_6^{4k} X[k], \quad 0 \le k \le 5$$

Therefore,

$$y[n] = x[((n-4))_6], \quad 0 \le n \le 5$$



(b) There are two ways to approach this problem. First, we attempt a solution by brute force.

$$\begin{aligned}
X[k] &= 4 + 3W_6^k + 2W_6^{2k} + W_6^{3k}, \quad W_6^k = e^{-j(2\pi k/6)} \text{ and } 0 \le k \le 5 \\
W[k] &= \mathcal{R}e\{X[k]\} \\
&= \frac{1}{2}\left(X[k] + X^*[k]\right) \\
&= \frac{1}{2}\left(4 + 3W_6^k + 2W_6^{2k} + W_6^{3k} + 4 + 3W_6^{-k} + 2W_6^{-2k} + W_6^{-3k}\right)
\end{aligned}$$

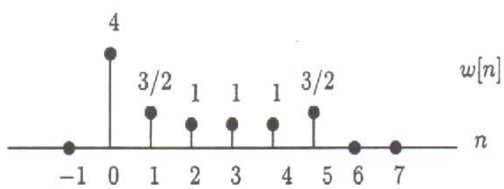
Notice that

$$
\begin{aligned}
W_N^k &= e^{-j(2\pi k/N)} \\
W_N^{-k} &= e^{j(2\pi k/N)} = e^{-j(2\pi/N)(N-k)} = W_N^{N-k} \\
W[k] &= 4 + \frac{3}{2}\left[W_6^k + W_6^{6-k}\right] + \left[W_6^{2k} + W_6^{6-2k}\right] + \frac{1}{2}\left[W_6^{3k} + W_6^{6-3k}\right], \quad 0 \le k \le 5
\end{aligned}
$$

So,

$$
\begin{aligned}
w[n] &= 4\delta[n] + \frac{3}{2}\Big(\delta[n-1] + \delta[n-5]\Big) + \delta[n-2] + \delta[n-4] \\
&\quad + \frac{1}{2}\Big(\delta[n-3] + \delta[n-3]\Big) \\
w[n] &= 4\delta[n] + \frac{3}{2}\delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] + \frac{3}{2}\delta[n-5], \quad 0 \le n \le 5
\end{aligned}
$$

Sketching $w[n]$:

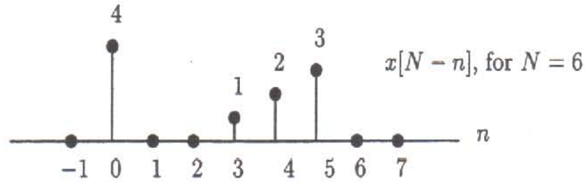


As an alternate approach, suppose we use the properties of the DFT as listed in Table 8.2.

$$
\begin{aligned}
W[k] &= \mathcal{R}e\{X[k]\} \\
&= \frac{X[k] + X^*[k]}{2} \\
w[n] &= \frac{1}{2}\ \mathrm{IDFT}\{X[k]\} + \frac{1}{2}\ \mathrm{IDFT}\{X^*[k]\} \\
&= \frac{1}{2}\Big(x[n] + x^*[((-n))_N]\Big)
\end{aligned}
$$

For $0 \le n \le N-1$ and $x[n]$ real:

$$
w[n] = \frac{1}{2}\Big(x[n] + x[N-n]\Big)
$$



So, we observe that $w[n]$ results as above.

(c) The DFT is decimated by two. By taking alternate points of the DFT output, we have half as many points. The influence of this action in the time domain is, as expected, the appearance of aliasing. For the case of decimation by two, we shall find that an additional replica of x[n] surfaces, since the sequence is now periodic with period 3.

From part (b):

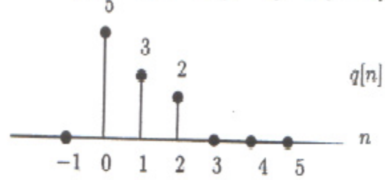
$$X[k] = 4 + 3W_6^k + 2W_6^{2k} + W_6^{3k}, \quad 0 \le k \le 5$$

Let $Q[k] = X[2k]$,

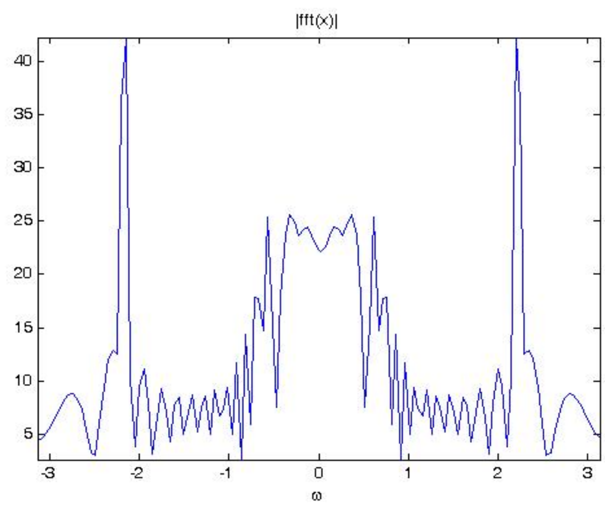
$$Q[k] = 4 + 3W_3^k + 2W_3^{2k} + W_3^{3k}, \quad 0 \le k \le 2$$

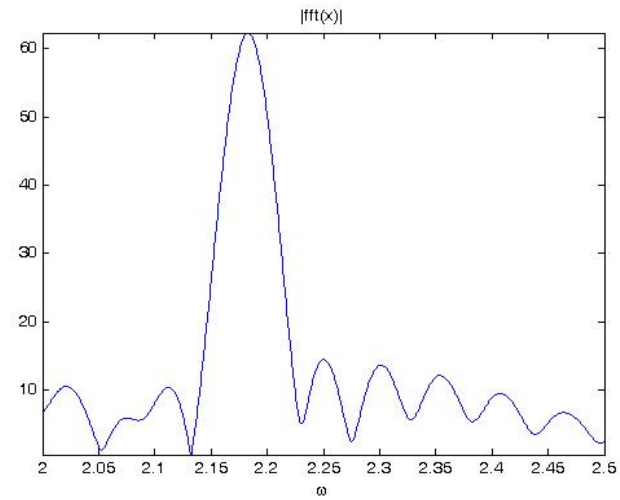
Noting that $W_3^{3k} = W_3^{0k}$

$$q[n] = 5\delta[n] + 3\delta[n-1] + 2\delta[n-2], \quad 0 \le n \le 2$$



Exercise 2




|fft(x)|
60
50
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20
10
2
2.05
2.1
2.15
2.2
2.25
2.3
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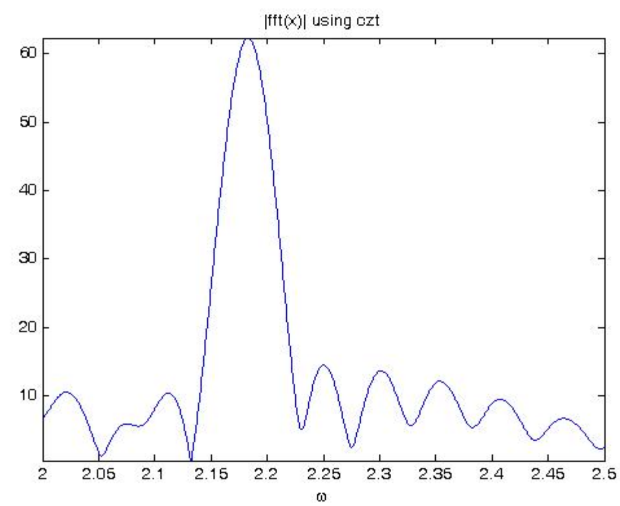
2.4
2.45
2.5
ω


|fft(x)| using czt
60
50
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10
2
2.05
2.1
2.15
2.2
2.25
2.3
2.35
2.4
2.45
2.5
ω