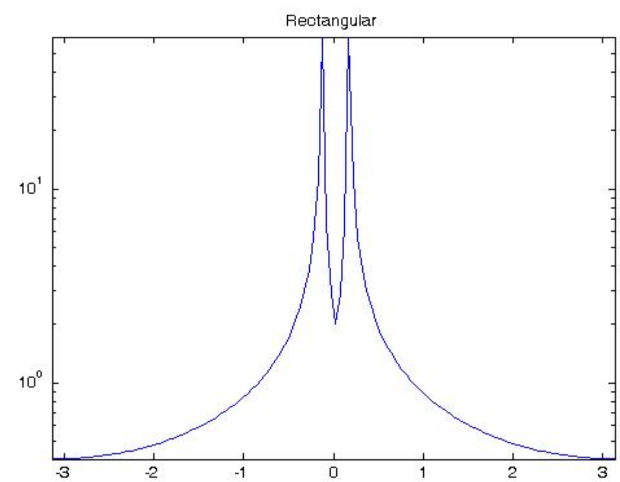
Home Project №5 - Solution

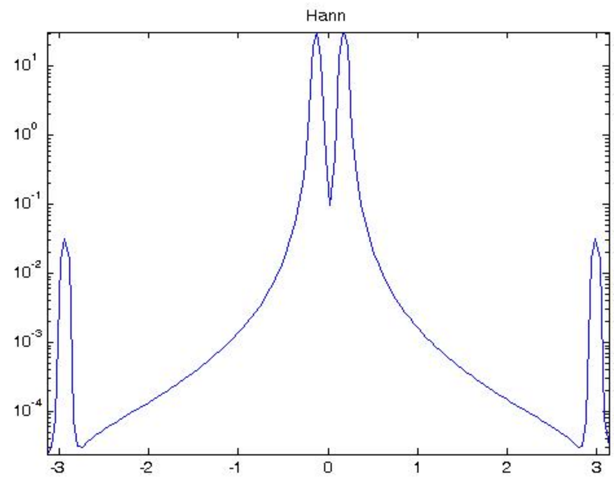
Exercise 1

1. $T_s \cdot 2\pi\Delta f = \frac{4\pi}{128+1} \Rightarrow \Delta f = 1.98Hz$.
2.

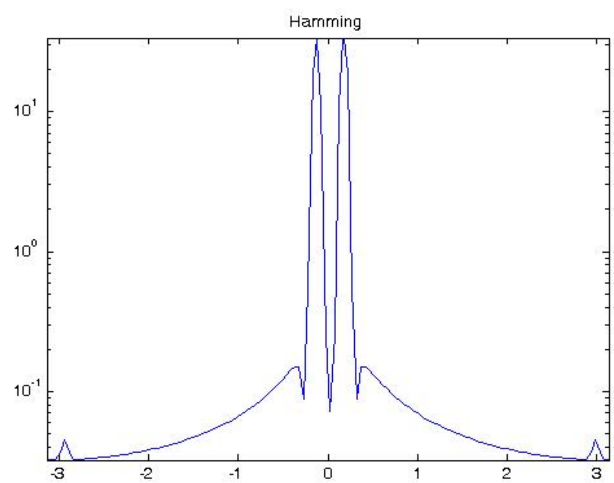


$0.1732/(2\pi \cdot T_s) \approx 3.5Hz$.
3. Hamming is expected to perform better, as its peak to side lobe ratio is greater.

4.

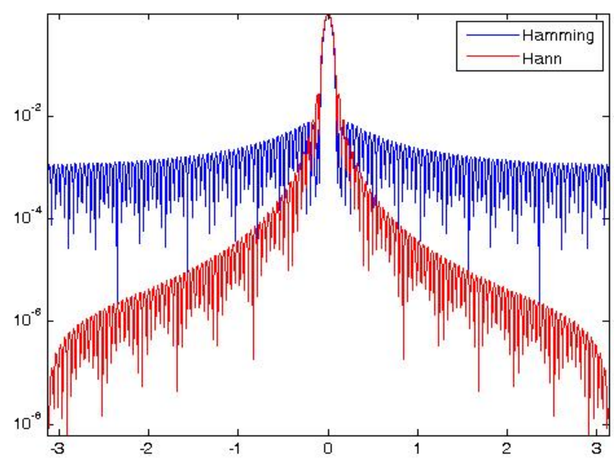


$0.1732/(2\pi \cdot T_s) \approx 3.5Hz.$
$2.993/(2\pi \cdot T_s) \approx 61Hz.$
5.



Same frequencies observed.
6. Hamming window was expected to perform better, however Hann window seems to reveal more of the additional frequency component then Hamming.

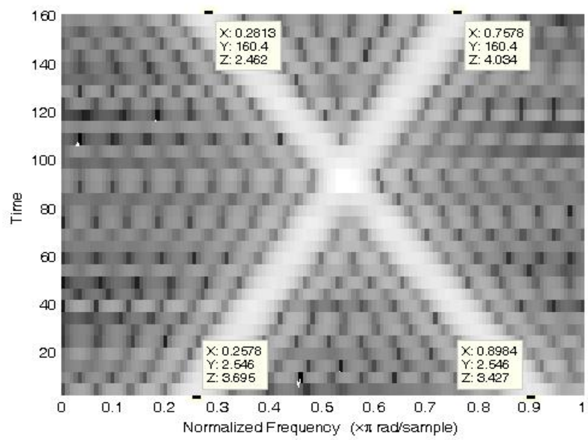
The reason is that Hann's peak to side lobe ratio decreases much more than Hamming's peak to side lobe ratio as the frequency increases (see figure below).



Exercise 2

$$S_c(t = nT) = \sum_{k=1}^{N} \sin(\omega_k nT + \frac{\lambda_k}{2}(nT)^2)$$

Two components appear, thus $N = 2$.



$T_s\omega_1 \approx 0.25\pi \Rightarrow \omega_1 \approx 2\pi \cdot 125 \frac{Rad}{sec}$
$T_s\omega_2 \approx 0.9\pi \Rightarrow \omega_2 \approx 2\pi \cdot 450 \frac{Rad}{sec}$

$T_s^2\lambda_1 \approx \frac{(0.75-0.25)\pi}{1024} \Rightarrow \lambda_1 \approx 1534 \frac{Rad}{sec^2}$
$T_s^2\lambda_2 \approx \frac{(0.3-0.9)\pi}{1024} \Rightarrow \lambda_2 \approx -1840 \frac{Rad}{sec^2}$

Exercise 3

(a) We must use the minimum specifications!

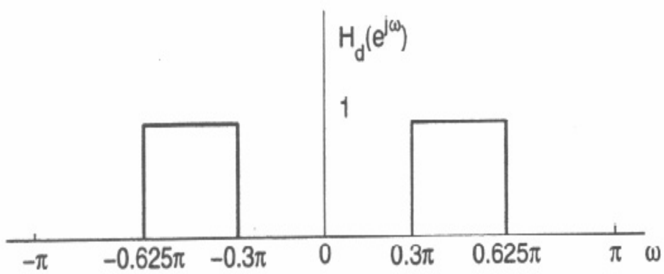
$$\begin{aligned} \delta &= 0.01 \\ \Delta\omega &= 0.05\pi \\ \mathrm{A} &= -20\log_{10}\delta = 40 \end{aligned}$$

$$M + 1 = \frac{\mathrm{A} - 8}{2.285\Delta\omega} + 1 = 90.2 \rightarrow 91$$

$$\beta = 0.5842(\mathrm{A} - 21)^{0.4} + 0.07886(\mathrm{A} - 21) = 3.395$$

(b) Since it is a linear phase filter with order 90, it has a delay of $90/2 = 45$ samples.

(c)



$$h_d[n] = \frac{\sin(.625\pi(n-45)) - \sin(.3\pi(n-45))}{\pi(n-45)}$$