University of Waterloo
Department of Electrical and Computer Engineering

ECE 413 – Digital Signal Processing
Midterm Exam, Spring 2007

June 13, 2007, 5:30-7:00 PM

Instructor: Dr. Oleg Michailovich

Student's name: ____

Student's ID #: ____

Instructions:

- This exam has **6** pages.
- **No books and lecture notes are allowed on the exam**. Please, turn off your cell phones, PDAs, etc., and place your bags, backpacks, books, and notes under the table or at the front of the room.
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- Answer all questions in the space provided on the exam paper. Use the back of pages if it is necessary.
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Problem №1 (25%)

By direct evaluation of the convolution sum, determine the output of a linear time-invariant system whose impulse response is given by $h[n] = a^{-n}u[-n]$, $0 < a < 1$, where $u[n]$ is the unit step sequence given by:

$$u[n] = \begin{cases} 1, & \text{for } n \geq 0, \\ 0, & \text{for } n < 0, \end{cases}$$

when the input $x[n]$ is equal to the unit step $u[n]$ defined above. **Hint**: The following formulas may be useful (for $|a| < 1$):

$$\sum_{k=-n}^{\infty} a^k = \frac{a^{-n}}{1-a} \quad \text{and} \quad \sum_{k=0}^{\infty} a^k = \frac{1}{1-a}.$$

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your answer is here ↘

Problem №2 (20%)

An LTI system has impulse response $h[n] = 5(1/2)^n u[n]$ where $u[n]$ is the unit step sequence defined in Problem 1. Use the (discrete-time) Fourier transform to find the output of this system when the input is $x[n] = (1/3)^n u[n]$. **Hints**:

A. The Fourier transform of $x[n] = a^n u[n]$ (with $|a| < 1$) is equal to $X^f(\theta) = \left(1 - a\, e^{-j\theta}\right)^{-1}$.

B. Try to use a factorization of the form:

$$\frac{b_1}{1 + a_1\, e^{-j\theta}} \cdot \frac{b_2}{1 - a_2\, e^{-j\theta}} = \frac{c_1}{1 + a_1\, e^{-j\theta}} + \frac{c_2}{1 - a_2\, e^{-j\theta}}.$$

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your answer is here ↘

Problem №3 (20%)

The continuous-time signal $x(t) = \sin(20\pi t) + \cos(40\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal

$$x[n] = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right).$$

A. Determine a choice of T consistent with this information.

B. Is your choice of T in Part A unique? If so, explain why. If not, specify another choice of T consistent with the information given.

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your answer is here ↘

Problem №4 (25%)

Consider the complex sequence:

$$x[n] = \begin{cases} e^{j\theta_0 n}, & 0 \le n \le N-1, \\ 0, & \text{otherwise.} \end{cases}$$

A. Find the Fourier transform $X^f(\theta)$ of $x[n]$.

B. Find the N-point DFT $X^d[k]$ of the finite-length sequence $x[n]$.

C. Find the DFT of $x[n]$ for the case $\theta_0 = 2\pi k_0/N$, where k_0 is an integer.

---

your answer is here ↘

Problem №5 (10%)

Let X_1 and X_2 be two $N \times N$ *circulant* matrices corresponding to finite-dimensional sequences $x_1[n]$ and $x_2[n]$ of length N, respectively. Find a general expression for the (k, n)-th element of matrix Y that is equal to the product of X_1 and X_2, viz. $Y = X_1 X_2$. **Hint**: How is Y related to signal $y[n] = \{x_1 \circledast x_2\}[n]$?

---

your answer is here ↘

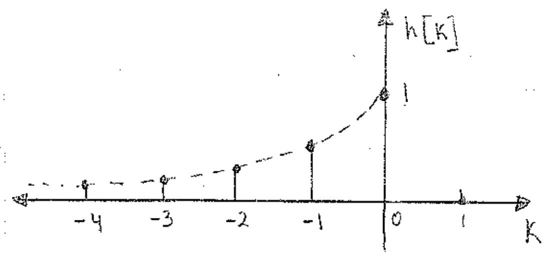
ECE 413
Midterm Solutions

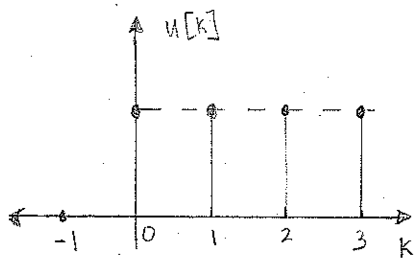
Problem 1

$h[n] = a^{-n} u[-n], \quad 0 < a < 1$

$$u[n] = \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases}$$

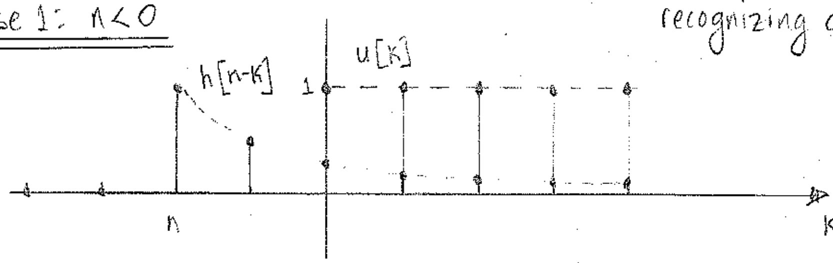
$y[n] = \sum_{K=-\infty}^{\infty} h[n-K]u[K]$ ← writing convolution formula down (10 marks)





Case 1: $n < 0$

recognizing cases (6 marks)



From the graph above : $y[n] = \sum_{K=0}^{\infty} h[n-K]u[K]$ (since $u[K]=0$, $K<0$)

choosing correct strategy (9 marks) → $= \sum_{K=0}^{\infty} \underbrace{a^{-(n-K)} u[-(n-K)]}_{h[n-K]} u[K]$
getting right answer

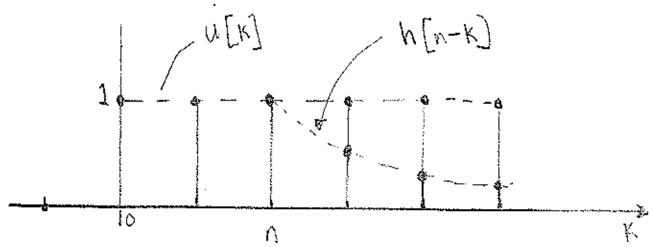
Note: $u[K-n] \cdot u[K] = u[K]$
(since $n < 0$)

$= \sum_{K=0}^{\infty} a^{K-n} u[K-n]\, u[K]$

$= \sum_{K=0}^{\infty} a^{K-n} u[K]$

$= \sum_{K=-n}^{\infty} a^{K} = \dfrac{a^{-n}}{1-a}$ (from hint)

Case 2: $n > 0$



From the graph above: $y[n] = \sum_{k=0}^{\infty} h[n-k]u[k]$ (since $u[k] = 0,\ k < 0$)

$$= \sum_{k=0}^{\infty} \underbrace{a^{-(n-k)} u[-(n-k)]}_{h[n-k]} u[k]$$

$$= \sum_{k=0}^{\infty} a^{k-n} u[k-n] u[k]$$

Note:
$u[k-n] \cdot u[k] = u[k-n]$
(since $n > 0$)

$$= \sum_{k=0}^{\infty} a^{k-n} u[k-n]$$

$$= \sum_{k=n}^{\infty} a^{k-n}$$

$$= \sum_{k=0}^{\infty} a^{k}$$

$$= \frac{1}{1-a} \quad \text{(from hint)}$$

Problem 2

$$h[n] = 5\left(-\tfrac{1}{2}\right)^n u[n] \iff H^f(\theta) = \frac{5}{1+\left(\frac{1}{2}\right)e^{-j\theta}} \quad \text{(from Hint A)} \quad (1)$$

$$x[n] = \left(\frac{1}{3}\right)^n u[n] \iff X^f(\theta) = \frac{1}{1-\left(\frac{1}{3}\right)e^{-j\theta}} \quad \text{(from Hint A)} \quad (2)$$

Now, $y[n] = h[n] * x[n]$ (3)

$\Rightarrow Y^f(\theta) = H^f(\theta)\, X^f(\theta)$ (4)

$\boxed{\text{10 marks}}$

$\boxed{\text{4 marks}}$ for correct substitution

Substituting (1) and (2) into (4):

$$Y^f(\theta) = \left(\frac{5}{1+\left(\frac{1}{2}\right)e^{-j\theta}}\right)\left(\frac{1}{1-\left(\frac{1}{3}\right)e^{-j\theta}}\right) \quad (5)$$

$\boxed{\text{1 mark}}$

$$= \frac{C_1}{1+\left(\frac{1}{2}\right)e^{-j\theta}} + \frac{C_2}{1-\left(\frac{1}{3}\right)e^{-j\theta}} \quad \text{(from hint)} \quad (6)$$

We need to find C_1 and C_2. From (5) and (6) it follows that for all θ:

$$C_1\left[1-\left(\frac{1}{3}\right)e^{-j\theta}\right] + C_2\left[1+\left(\frac{1}{2}\right)e^{-j\theta}\right] = 5 \quad (7)$$

substituting $\theta = 0$ into (7):

$$C_1\left(1-\frac{1}{3}\right) + C_2\left(1+\frac{1}{2}\right) = 5$$

$$\Rightarrow \frac{2C_1}{3} + \frac{3C_2}{2} = 5 \quad (8)$$

Similarly, for $\theta = \pi$:

$$c_1\left(1+\frac{1}{3}\right) + c_2\left(1-\frac{1}{2}\right) = 5$$

$$\frac{4c_1}{3} + \frac{c_2}{2} = 5 \qquad (9)$$

Solving (8) and (9) for c_1 & c_2 :

$$\frac{4c_1}{3} + 3c_2 = 10 \qquad (\text{Eq. } 8) \times 2$$

$$-\quad \frac{4c_1}{3} + \frac{c_2}{2} = 5$$

$$\frac{5c_2}{2} = 5$$

$$c_2 = 2 \qquad (10)$$

Substitute (10) into (9) or (8) : $\boxed{3 \text{ marks}}$ for correct c_1 & c_2

$$\frac{4c_1}{3} + \frac{2}{2} = 5$$

$$c_1 = 3. \qquad (11)$$

Therefore from (10), (11), (6) and Hint A, $\boxed{2 \text{ marks}}$

$$y[n] = 3\left(-\frac{1}{2}\right)^n u[n] + 2\left(\frac{1}{3}\right)^n u[n]$$

Problem 3

$$x(t) = \sin(20\pi t) + \cos(40\pi t)$$

$$x[n] = x(nT_s) = \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

Part A

Suppose $T_s = \frac{1}{100}$. Then clearly:

$$x(nT_s) = \sin(20\pi n T_s) + \cos(40\pi n T_s)$$

$$= \sin\left(\frac{\pi n}{5}\right) + \cos\left(\frac{2\pi n}{5}\right)$$

So an obvious choice of T_s is $T_s = \frac{1}{100}$ ▨ $\boxed{\text{10 marks}}$

Part B

First we determine the period of $x(t)$. Note that the $\sin(20\pi t)$ term has period $T_1 = 0.1$ $\left(\frac{2\pi}{T_1} = 20\pi\right)$. The $\cos(40\pi t)$ term has period $T_2 = 0.05$ $\left(\frac{2\pi}{T_2} = 40\pi\right)$. Therefore the period of $x(t)$ is the least common multiple of T_1 & T_2; the period must be $T = 0.1$.

So we have: $x(t) = x(t + KT)$ $\forall K$, $T = 0.1$

Therefore $x(nT_s) = x(nT_s + KT)$ $\forall K$

Now suppose we sample at $T_s' = T_s + mT$, where m is some integer, and $T = 0.1$ (the period of $x(t)$)

Then $x(nT_s') = x(n(T_s + mT))$

$$= x(nT_s + nmT)$$

$$= x(nT_s)$$

Therefore T_s is clearly not unique. Any sampling period $T_s = \frac{1}{100} + K(0.1)$ is consistent with the information given ∎

$\boxed{5 \text{ marks}}$ for saying "T_s is not unique"
$\boxed{5 \text{ marks}}$ for providing an alternative T_s

If your answer was incorrect, you recieved up to $\boxed{5 \text{ marks}}$ for mentioning the word "periodic" in your explanation.

Problem 4

$$x[n] = \begin{cases} e^{j\theta_0 n}, & 0 \le n \le N-1 \\ 0, & \text{otherwise.} \end{cases}$$

$$X^f(\theta) = \sum_{n=0}^{N-1} x[n]\, e^{-j\theta n}$$

$$= \sum_{n=0}^{N-1} e^{-j(\theta-\theta_0)n}$$

$\boxed{7.5 \text{ marks}}$ for getting here

$\boxed{2.5 \text{ marks}}$ for geometric series formula (anywhere)

$$= \frac{1 - e^{-j(\theta-\theta_0)N}}{1 - e^{-j(\theta-\theta_0)}}$$

(since we have the sum of a geometric series) ⊛

$$= \frac{\exp\left(-j\frac{(\theta-\theta_0)N}{2}\right)\left(\exp\left(j\frac{(\theta-\theta_0)N}{2}\right) - \exp\left(-j\frac{(\theta-\theta_0)N}{2}\right)\right)}{\exp\left(-j\frac{(\theta-\theta_0)}{2}\right)\left(\exp\left(j\left(\frac{\theta-\theta_0}{2}\right)\right) - \exp\left(-j\left(\frac{\theta-\theta_0}{2}\right)\right)\right)}$$

$$= \exp\left(-j\frac{(\theta-\theta_0)(N-1)}{2}\right) \frac{\sin\left(\frac{(\theta-\theta_0)N}{2}\right)}{\sin\left(\frac{\theta-\theta_0}{2}\right)}$$

Part B

$$X^d[k] = X^f\left(\frac{2\pi k}{N}\right) = \exp\left(-j\,0.5\left(\frac{2\pi}{N}k - \theta_0\right)(N-1)\right) \frac{\sin\left(0.5\left(\frac{2\pi k}{N} - \theta_0\right)N\right)}{\sin\left(0.5\left(\frac{2\pi k}{N} - \theta_0\right)\right)}$$

$\boxed{4 \text{ marks}}$ for correct formula of DFT

$\boxed{1 \text{ mark}}$ for answer in simplified form (as in ⊛)

or $\boxed{5 \text{ marks}}$ → for recognizing $X^d[k] = X^f\left(\frac{2\pi k}{N}\right)$

Part C

Suppose $\theta_0 = \frac{2\pi k_0}{N}$, where k_0 is an integer.

Then,

$$X^d[k] = \exp\left(-j0.5\left(\frac{2\pi k}{N} - \frac{2\pi k_0}{N}\right)(N-1)\right) \frac{\sin\left(0.5\left(\frac{2\pi k}{N} - \frac{2\pi k_0}{N}\right)N\right)}{\sin\left(0.5\left(\frac{2\pi k}{N} - \frac{2\pi k_0}{N}\right)\right)}$$

We have two cases, $k \neq k_0$ and $k = k_0$.

Case 1: $k \neq k_0$

The sin term in the numerator is zero. The denominator is non-zero.

$\therefore X^d[k] = 0$

Case 2: $k = k_0$

Notice the ratio of sin terms can be expressed as follows:

$$\frac{\sin\left(0.5\left(\frac{2\pi k}{N} - \frac{2\pi k_0}{N}\right)N\right)}{\sin\left(0.5\left(\frac{2\pi k}{N} - \frac{2\pi k_0}{N}\right)\right)} = \frac{\sin\left(\pi(k-k_0)\right)}{\sin\left(\pi\frac{(k-k_0)}{N}\right)} \cdot \frac{\pi\left(\frac{k-k_0}{N}\right)}{\pi(k-k_0)} \cdot N$$

$$= \frac{\operatorname{sinc}(k-k_0)}{\operatorname{sinc}\left(\frac{k-k_0}{N}\right)} \cdot N$$

Therefore $X^d[k_0] = \frac{\text{sinc}(0)}{\text{sinc}(0)} \cdot N$

$= N$

The final solution is :

$$X^d[k] = \begin{cases} 0, & k \neq k_0 \\ N, & k = k_0 \end{cases}$$

Alternate (easier) solution to part c

$$X^d[k] = \sum_{n=0}^{N-1} e^{j\theta_0 n} W_N^{-nk} \leftarrow \boxed{6 \text{ marks}}$$ for stating formula

$$= \sum_{n=0}^{N-1} W_N^{k_0 n} \, W_N^{-nk}$$

$$= \sum_{n=0}^{N-1} W_N^{-n(k-k_0)} \quad \boxed{7.5 \text{ marks}}$$ for getting to this form

$$= N\delta\left[(k-k_0) \bmod N\right]$$

$\boxed{2.5 \text{ marks}}$ for this answer.

Problem 5

We know $y[n] = \{x_1 \circledast x_2\}[n]$

Also; $Y = X_1 X_2$, where X_1 and X_2 are circulant matrices corresponding to $x_1[n]$ and $x_2[n]$.

We now prove that Y must be the circulant matrix corresponding to $y[n]$.

Suppose we have a column vector Z corresponding to a sequence $z[n]$.

Let Y' be the circulant matrix corresponding to $y[n]$.

Let $w[n] = \{y \circledast z\}[n]$

$= \{x_1 \circledast x_2 \circledast z\}[n]$

Therefore $W = X_1 X_2 Z$ and $W = Y' Z$, where W is the column vector corresponding to $w[n]$.

So $Y' = X_1 X_2 = Y$

Therefore Y must be the circulant matrix corresponding to $y[n]$.

Hence $Y_{k,n} = y[(k-n) \bmod N]$ ▨

10 marks for correct final answer
or 9 marks for stating Y is the circulant corresponding to $y[n]$
or 8 marks for recognizing relationship between convolution and multiplication by circulant
or 7 marks for knowing general form of circulant matrix + painful calculations
or 6 marks for painful calculations

University of Waterloo
Department of Electrical and Computer Engineering

ECE 413 – Digital Signal Processing
Final Exam, Spring 2007

August 4, 2007, 12:30-3:00 PM

Instructor: Dr. Oleg Michailovich

Student's name: ____

Student's ID #: ____

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Problem №1 (18%)

The linear filter $h[n]$ with transfer function

$$H^f(\theta) = \begin{cases} -\jmath, & 0 < \theta < \pi, \\ \jmath, & -\pi < \theta < 0, \end{cases}$$

is used to generate signal $y[n]$, such that $\Re\{y[n]\} = x[n]$ and $\Im\{y[n]\} = \{x * h\}[n]$ (with $\Re$ and $\Im$ denoting the real and imaginary parts, respectively, viz. $y[n] = \Re\{y[n]\} + \jmath\Im\{y[n]\}$) for some input signal $x[n]$. Given that the Fourier transform $X^f(\theta)$ of $x[n]$ is defined as

$$\Re\{X^f(\theta)\} = \begin{cases} 0, & -\pi \le \theta < -\theta_0, \\ \frac{\theta}{\theta_0} + 1, & -\theta_0 \le \theta < 0, \\ 1 - \frac{\theta}{\theta_0}, & 0 \le \theta < \theta_0, \\ 0, & \theta_0 \le \theta \le \pi, \end{cases} \qquad \text{and} \qquad \Im\{X^f(\theta)\} = 0, \quad -\pi \le \theta \le \pi,$$

for some $0 < \theta_0 < \pi$, determine and sketch $Y^f(\theta)$.

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your answer is here ↘

Problem №2 (23%)

When the input to an LTI system is

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n-1],$$

the output is

$$y[n] = 6\left(\frac{1}{2}\right)^n u[n] - 6\left(\frac{3}{4}\right)^n u[n],$$

when $u[n]$ is the unit-step function.

A. Find the transfer function $H(z)$ of the system. Plot the poles and zeros of $H(z)$, and indicate the region of convergence.

B. Find the impulse response $h[n]$ of the system.

C. Write the difference equation that characterizes the system.

D. If the system stable? Is it causal?

---

your answer is here ↘

Problem №3 (18%)

Consider a stable discrete-time signal $x[n]$ whose discrete-time Fourier transform $X^f(\theta)$ satisfies the equation

$$X^f(\theta) = X^f(\theta - \pi),$$

and has even symmetry, i.e., $x[n] = x[-n]$.

A. Show that $X^f(\theta)$ is periodic with a period π.

B. Find the value of $x[3]$. (Hint: Find values of all odd-indexed samples.)

C. Let $y[n]$ be a decimated version of $x[n]$, viz. $y[n] = x[2n]$. Can you reconstruct $x[n]$ from $y[n]$ for all n? If yes, how? If no, justify your answer.

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your answer is here ↘

Problem №4 (23%)

The transfer function of a discrete-time system is

$$H(z) = \frac{2}{1 - e^{-0.2} z^{-1}} - \frac{1}{1 - e^{-0.4} z^{-1}}. \tag{1}$$

Assume that $H(z)$ was obtained by the bilinear transform method with $T = 2$. Find the (continuous-time) transfer function $H^S(s)$ that could be the basis for the design. Is your answer unique? If not, find another $H^S(s)$.

---

your answer is here ↘

Problem №5 (18%)

Consider the following ideal frequency response for a multi-band filter:

$$H_d(\theta) = \begin{cases} e^{-\jmath\theta N/2}, & 0 \le |\theta| \le 0.3\pi, \\ 0, & 0.3\pi < |\theta| < 0.6\pi, \\ 0.5e^{-\jmath\theta N/2}, & 0.6\pi \le |\theta| \le \pi. \end{cases}$$

The impulse response $h_d[n]$ is multiplied by a Hamming window with $N = 51$, resulting in a linear-phase FIR filter with impulse response $h[n]$.

A. What is the delay of the filter $h[n]$?

B. Determine the ideal impulse response $h_d[n]$.

---

your answer is here ↘

Problem #1

Note that $X(e^{j\omega})$ is real, and $Y(e^{j\omega})$ is given by:

$$Y(e^{j\omega}) = \begin{cases} -jX(e^{j\omega}) & , \quad 0 < \omega < \pi \\ +jX(e^{j\omega}) & , \quad -\pi < \omega < 0. \end{cases}$$

$w[n] = x[n] + jy[n]$, therefore:

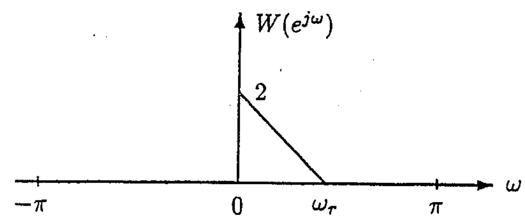
$$W(e^{j\omega}) = X(e^{j\omega}) + jY(e^{j\omega}).$$

Using the above, we get:

$$jY(e^{j\omega}) = \begin{cases} X(e^{j\omega}) & , \quad 0 < \omega < \pi \\ -X(e^{j\omega}) & , \quad -\pi < \omega < 0. \end{cases}$$

We thus conclude:

$$W(e^{j\omega}) = \begin{cases} 2X(e^{j\omega}) & , \quad 0 < \omega < \pi \\ 0 & , \quad -\pi < \omega < 0. \end{cases}$$



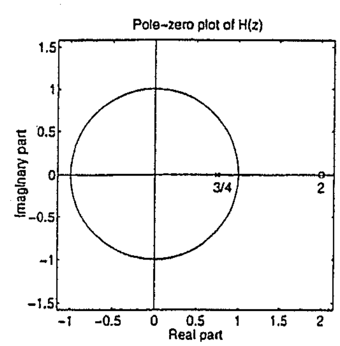
Problem #2

(a)

$$X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}, \qquad \frac{1}{2} < |z| < 2$$

$$Y(z) = \frac{6}{1-\frac{1}{2}z^{-1}} - \frac{6}{1-\frac{3}{4}z^{-1}}, \qquad |z| > \frac{3}{4}$$

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{\frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-\frac{3}{4}z^{-1})}}{\frac{-\frac{3}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}} \\ &= \frac{1-2z^{-1}}{1-\frac{3}{4}z^{-1}}, \qquad |z| > \frac{3}{4} \end{aligned}$$



(b)

$$h[n] = \left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{3}{4}\right)^{n-1} u[n-1]$$

(c)

$$y[n] - \frac{3}{4}y[n-1] = x[n] - 2x[n-1]$$

(d) The system is stable because the ROC includes the unit circle. It is also causal since $h[n] = 0$ for $n < 0$.

Problem #3

(a) Since $X(e^{j\omega}) = X(e^{j(\omega-\pi)})$, $X(e^{j\omega})$ is periodic with period π.

(b) Using the inverse DTFT,

$$\begin{aligned} x[n] &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j(\omega-\pi)})e^{j\omega n}d\omega \\ &= \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j(\omega+\pi)n}d\omega \\ &= \frac{1}{2\pi}e^{j\pi n}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}d\omega \\ &= (-1)^n x[n]. \end{aligned}$$

All odd samples of $x[n] = 0$, because $x[n] = -x[n]$. Hence $x[3] = 0$.

(c) Yes, $y[n]$ contains all even samples of $x[n]$, and all odd samples of $x[n]$ are 0.

$$x[n] = \begin{cases} y[n/2], & n \text{ even} \\ 0, & \text{otherwise} \end{cases}$$

Problem #4

Using the inverse relationship for the bilinear transform,

$$z = \frac{1+(T_d/2)s}{1-(T_d/2)s}$$

we get

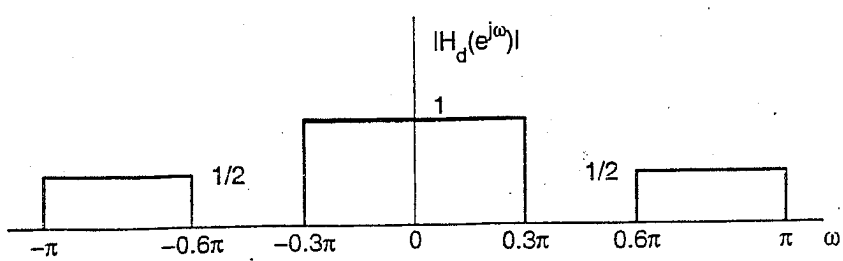
$$\begin{aligned} H_c(s) &= \frac{2}{1-e^{-0.2}\left(\frac{1-s}{1+s}\right)} - \frac{1}{1-e^{-0.4}\left(\frac{1-s}{1+s}\right)} \\ &= \frac{2(s+1)}{s(1+e^{-0.2})+(1-e^{-0.2})} - \frac{(s+1)}{s(1+e^{-0.4})+(1-e^{-0.4})} \\ &= \left(\frac{2}{1+e^{-0.2}}\right)\left(\frac{s+1}{s+\frac{1-e^{-0.2}}{1+e^{-0.2}}}\right) - \left(\frac{1}{1+e^{-0.4}}\right)\left(\frac{s+1}{s+\frac{1-e^{-0.4}}{1+e^{-0.4}}}\right) \end{aligned}$$

Since the bilinear transform does not introduce any ambiguity, the representation is unique.

Problem #5

(a) The delay is $\frac{M}{2} = 24$.

(b)



This can be viewed as the sum of two lowpass filters, one of which has been shifted in frequency (modulation in time-domain) to $\omega = \pi$. The linear phase factor adds a delay.

$$h_d[n] = \frac{\sin(0.3\pi(n-24))}{\pi(n-24)} + \frac{1}{2}(-1)^{(n-24)}\frac{\sin(0.4\pi(n-24))}{\pi(n-24)}$$