Home Project №2

Due on June 12, 2009

Exercise 1

A continuous-time signal $x(t)$ is passed through a filter with impulse response $h(t)$ and then sampled at interval T; see Figure 1(A). The signal $x(t)$ is band-limited to $\pm\Omega_1$, and the Fourier transform of the filter is band-limited to $\pm\Omega_2$. We wish to change the order of operations: sample the signal $x(t)$ first and then pass the sampled signal through a digital filter; see Figure 1(B). We require that:

- The impulse response of the digital filter be $T\,h(nT)$;
- The outputs of the two systems be equal for any input signal $x(t)$ that meets the bandwidth restriction.

What is the condition on the sampling interval T to meet the above requirement?

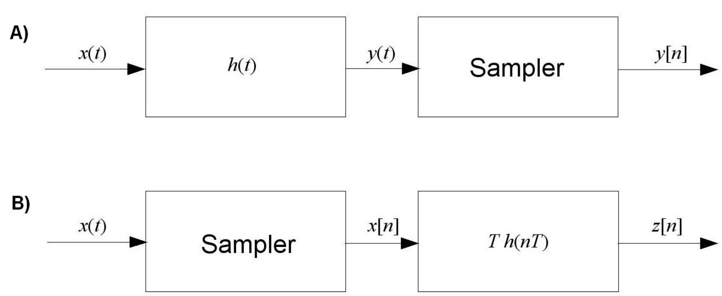

Figure 1: Block diagrams

Exercise 2

A continuous-time signal $x_c(t)$ is is passed though a filter with impulse response $h_c(t)$ as depicted in Figure 2(a).The signal $x_c(t)$ is band-limited to $\pm\Omega_1$, and the Fourier transform of the filter is band-limited to $\pm\Omega_2$. We wish to implement the filtering of Figure 2(a) in the discrete domain, as depicted in Figure 2(b). We require that:

- The impulse response of the digital filter be $h[n] = Th_c(nT)$;
- The outputs of the two systems be equal for any input signal $x_c(t)$ that meets the bandwidth restriction.

When the above requirements are met, the discrete system is said to be an *impulse invariant* version of its continuous counterpart $h_c(t)$. How should one choose the sampling interval T to meet this requirement?

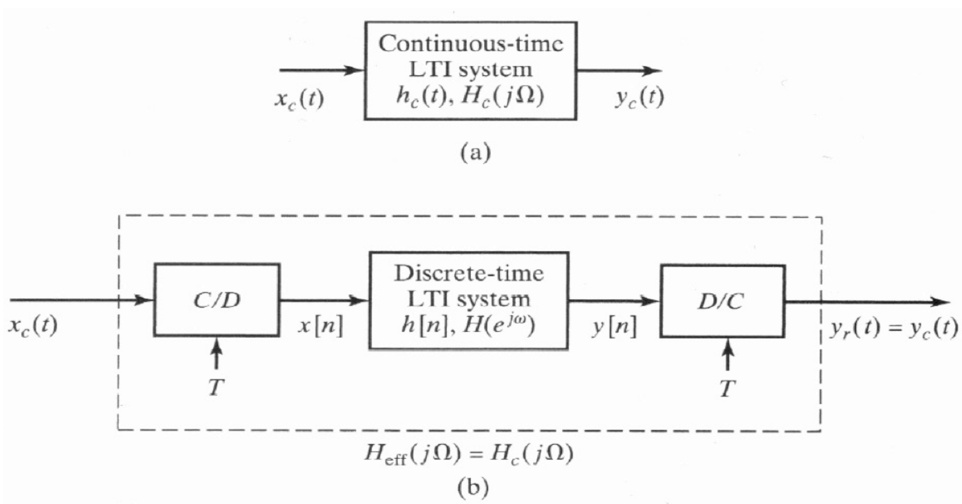

Figure 2: A continuous-time system and its discrete-time equivalent system.

Exercise 3

A continuous-time signal $x_c(t)$ is sampled and filtered by a discrete low pass filter $h[n]$. The signal $x_c(t)$ is band-limited to $\pm\Omega_1$, and the discrete-time

Fourier transform (DTFT) of the filter is given by:

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \pi/L, \\ 0 & \pi/L < |\omega| < \pi \end{cases} \tag{1}$$

Find the *lowest* sampling frequency that would result in zero aliasing error in the filtered discrete signal.

Exercise 4

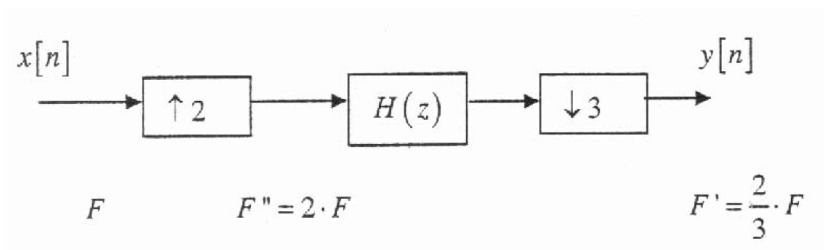

Figure 3: Rational factor decimation system

The above figure depicts a system that is used for decimating $x[n]$ using a rational factor of 2/3. This type of system is commonly used for changing the sampling rate of audio signals. To implement the given system in Matlab use the following Matlab functions:

- To implement the compressor and expander use `downsample` and `upsample` functions, respectively.
- To create a low-pass filter with a cutoff of π/L, use the command `h = fir1(128,1/L)`.
- To perform convolution use `conv.m`.

To apply the system to the audio signal "sound.wav" use the following Matlab functions:

- To load the signal to your Matlab workspace use `wavread.m`.

- To listen to the audio signal use `sound.m`. Make sure you are using the appropriate sampling frequency before and after you decimate the signal.

Listen to the signal using $L = 2, 3$. Which sounds are clearer and more similar to the original sound? Explain why.