# PMATH 352 — Assignment 1, due Friday Jan 23

1. Express the following in the form $a + bi$:

   i) $(2+3i)/(4+5i)$

   ii) $(1-i)^{33}$

   iii) $(11+3i)(11-3i)+(3+11i)(3-11i)$

2. Find all complex numbers $z$ that satisfy $z^5 = 32i$ (express your solutions in polar form).

3. Use de Moivre's theorem to give formulas for $\cos(5\theta)$ and $\sin(5\theta)$ in terms of $\cos(\theta)$ and $\sin(\theta)$.

4. Prove that $\sum_0^N \cos(n\theta) = 1/2 + \sin\left((N+1/2)\theta\right)/(2\sin(\theta/2))$.

5. Show, for all $w, z \in \mathbb{C}$, that: $|z - \bar{z}w|^2 - |z-w|^2 = (1-|z|^2)(1-|w|^2)$. Use it to prove: if $|w|, |z| < 1$, then $|z-w|/|1-\bar{z}w| < 1$.

6. Find the images of the following under the transformation $w = 1/z$: a) the circle $|z-1| = 1$, b) the circle $|z-1/2| = 3$, c) the line $\Re z = 4$.

7. Find a Möbius transformation that sends 3 to 4, 4 to $i$, and $i$ to 3.

8. Let $\mathbb{H} = \{z \in \mathbb{C} : \Im z > 0\}$ denote the upper half plane. Show, if $a, b, c, d \in \mathbb{R}$, and $ad - bc > 0$, that $f(z) = (az+b)/(cz+d)$ maps $\mathbb{H}$ onto $\mathbb{H}$.

9. Let $a_m, b_m$ be complex numbers and let $\sum_{m=0}^\infty a_m = A$, $\sum_{m=0}^\infty b_m = B$ be two convergent sums. Assume further that the first sum converges absolutely. Let $c_m = \sum_{k=0}^m a_{m-k}b_k$. Show that $\sum_{m=0}^\infty c_m = AB$, i.e. show that the sequence of partial sums of the $c_m$'s converges to $AB$.

10. Starting with $\exp(z) = \sum_{m=0}^\infty z^m/m!$, show that $\exp(z+w) = \exp(z)\exp(w)$ as an identity of power series. Justify your steps.