**Binary expansion of a real number.**

Let $r$ be a real number, with $0 \leq r \leq 1$. Let

$$r_1 = \max\{k \in \{0,1\} = \mathbb{Z}_2 : k/2 \leq r\}$$

and put $s_1 = r_1/2$.

Now proceed recursively. Assume we have defined $r_1, r_2, \ldots, r_n$ and put $s_n = \sum_{i=1}^{n} r_i/2^i$. Let

$$r_{n+1} = \max\left\{k \in \mathbb{Z}_2 : k/2^{n+1} \leq r - s_n\right\}$$

and put

$$s_{n+1} = \sum_{i=1}^{n+1} r_i/2^i.$$

Then the sequence $(s_n)_{n=1}^{\infty}$ converges and its limit is

$$\lim_{n\to\infty} s_n = r.$$