Fix complex Euclidean spaces $\mathcal{X}$ and $\mathcal{Y}$. Suppose we want to find a channel $\Phi \in C(\mathcal{X}, \mathcal{Y})$ which maximizes the sum of some functions $F_1, \dots, F_n : C(\mathcal{X}, \mathcal{Y}) \to \mathbb{R}$. Suppose further that, for fixed $\Phi \in C(\mathcal{X}, \mathcal{Y})$, each $F_k(\Phi)$ can be expressed as the optimal solution to an SDP whose primal and dual both have Slater points. Suppose further that for these SDPs, only the B component (that is, the right-hand side of the constraint) depends on Φ. In other words, for each k, there exist spaces $\mathcal{W}_k, \mathcal{Z}_k$, an objective function given by $C_k \in \mathcal{W}_k$, a constraint function $\Psi_k \in \mathcal{L}(\mathcal{W}_k, \mathcal{Z}_k)$, and an affine mapping $B_k : C(\mathcal{X}, \mathcal{Y}) \to L(\mathcal{Z}_k)$ producing the following SDP for Φ:

$$
\begin{array}{lrl}
(P_k(\Phi)) \qquad & \max & \langle C_k, X_k \rangle \\
& \text{subject to} & \Psi_k(X_k) = B_k(\Phi) \\
& & X_k \in \operatorname{Pos}(\mathcal{W}_k)
\end{array}
$$

We note that the dual of this SDP is:

$$
\begin{array}{lrl}
(D_k(\Phi)) \qquad & \min & \langle B_k(\Phi), Y_k \rangle \\
& \text{subject to} & \Psi_k^*(Y_k) \succeq C_k \\
& & Y_k \in \operatorname{Herm}(\mathcal{Z}_k)
\end{array}
$$

Express each $B_k(\Phi)$ as $A_k(\Phi) + D$, for $A_k \in L(C(\mathcal{X}, \mathcal{Y}), L(\mathcal{Z}_k))$ and $D \in L(\mathcal{Z}_k)$. The complementary slackness condition for $(P_k(\Phi))$ and $(D_k(\Phi))$ is:

$$
(CS_k) \qquad X_k \cdot (\Psi_k^*(Y_k) - C_k) = 0
$$

Suppose we have an easy way of finding an optimal solution to $(D_k(\Phi))$. That is, we have an easily computed $f : C(\mathcal{X}, \mathcal{Y}) \to \operatorname{Herm}(\mathcal{Z}_k)$ such that $f(\Phi)$ is always an optimal solution for $(D_k(\Phi))$.

By replacing Φ with its Choi representation $M = J(\Phi)$, we can express the original problem as an SDP:

$$
\begin{array}{lrll}
(P_\Sigma) \qquad & \max & \displaystyle\sum_{k=1}^{n} \langle C_k, X_k \rangle & \\
& \text{subject to} & \Psi_k(X_k) - A_k(M) = D & \forall k \\
& & \operatorname{Tr}_{\mathcal{Y}}(M) = \mathbb{1}_{\mathcal{X}} & \\
& & X_k \succeq 0 & \forall k \\
& & M \succeq 0 &
\end{array}
$$

The dual of this SDP is:

$$
\begin{array}{lrll}
(D_\Sigma) \qquad & \min & \operatorname{Tr}(Z) + \displaystyle\sum_{k=1}^{n} \langle D_k, Y_k \rangle & \\
& \text{subject to} & \Psi_k^*(Y_k) \succeq C_k & \forall k \\
& & \mathbb{1}_{\mathcal{Y}} \otimes Z \succeq \displaystyle\sum_{k=1}^{n} A_k^*(Y_k) & \\
& & Y_k \in \operatorname{Herm}(\mathcal{Z}_k) & \forall k \\
& & Z \in \operatorname{Herm}(\mathcal{X}) &
\end{array}
$$