─── MODULE *musical_chairs* ───

EXTENDS *Integers*, *FiniteSets*

CONSTANTS *CHAIRS*, *PLAYERS*

VARIABLES
    activePlayers, *activeChairs*,
    occupied, *music_playing*, *state*

$vars \triangleq \langle activePlayers,\ activeChairs,\ occupied,\ music\_playing,\ state \rangle$

$STATE\_ \triangleq \{\text{"Start"},\ \text{"Walking"},\ \text{"Sitting"},\ \text{"Won"}\}$

Helper predicate for range of a function
$Range(f) \triangleq \{f[x] : x \in \text{DOMAIN } f\}$

Typing invariant
$TypeOK \triangleq$
    $\wedge\quad activePlayers\ \subseteq PLAYERS$
    $\wedge\quad activeChairs\ \ \subseteq CHAIRS$
    $\wedge\quad occupied \in [activeChairs \rightarrow activePlayers] \cup \{\langle\rangle\}$
    $\wedge\quad music\_playing \in \text{BOOLEAN}$ whether music is playing
    $\wedge\quad state \in STATE\_$

Initial state
$Init \triangleq$
    $\wedge activePlayers = PLAYERS$ force all *activePlayers* and
    $\wedge activeChairs = CHAIRS$ *activeChairs* to be included
    $\wedge Cardinality(activePlayers) > 1$
    $\wedge Cardinality(activePlayers) = Cardinality(activeChairs) + 1$
    $\wedge occupied = \langle\rangle$ initially the empty function
    $\wedge music\_playing \in \text{BOOLEAN}$
    $\wedge state = \text{"Start"}$

$Walk \triangleq$
    $\wedge state = \text{"Start"}$

1

$\land$ *music_playing*
$\land$ *Cardinality*(*activePlayers*) > 1
$\land$ *occupied′* = ⟨⟩
$\land$ *state′* = "Walking"
$\land$ UNCHANGED ⟨*activeChairs*, *activePlayers*, *music_playing*⟩

Sit $\triangleq$
    $\land$ *state* = "Walking"
    $\land$ ¬*music_playing*
    $\land$ *occupied′* ∈ [*activeChairs* → *activePlayers*]
    each chair maps to only one player
    $\land$ ∀ *c* ∈ *activeChairs*, *p1*, *p2* ∈ *activePlayers* :
        occupied′[*c*] = *p1* ∧ *occupied′*[*c*] = *p2* ⇒ *p1* = *p2*
    each occupying player sits on one chair
    $\land$ ∀ *p* ∈ *Range*(*occupied′*), *c1*, *c2* ∈ DOMAIN *occupied′* :
        occupied′[*c1*] = *p* ∧ *occupied′*[*c2*] = *p* ⇒ *c1* = *c2*
    there's a player that didn't get to sit down
    $\land$ ∃ *p* ∈ *activePlayers*: *p* ∉ *Range*(*occupied′*)
    $\land$ *state′* = "Sitting"
    $\land$ UNCHANGED ⟨*activeChairs*, *activePlayers*, *music_playing*⟩

EliminateLoser $\triangleq$
    $\land$ *state* = "Sitting"
    $\land$ *Cardinality*(*activePlayers*) > 1
    $\land$ *Cardinality*(*activePlayers*) − *Cardinality*(*Range*(*occupied*)) = 1
    $\land$ *activePlayers′* = *Range*(*occupied*)
    $\land$ *activeChairs′* = *activeChairs* \ {CHOOSE *c* ∈ *activeChairs* : TRUE}
    $\land$ *Cardinality*(*activeChairs′*) = *Cardinality*(*activeChairs*) − 1
    $\land$ *occupied′* = ⟨⟩
    $\land$ *state′* = "Start"
    $\land$ UNCHANGED *music_playing*

Win $\triangleq$
    $\land$ *state* = "Sitting"
    $\land$ *Cardinality*(*activePlayers*) = 1
    $\land$ *state′* = "Won"

ChangeMusicPlaying $\triangleq$
    $\land$ *music_playing′* ∈ BOOLEAN
    $\land$ UNCHANGED ⟨*activeChairs*, *activePlayers*, *state*, *occupied*⟩

Safety invariants

OneMorePlayerThanChairs $\triangleq$
    Cardinality(*activePlayers*) = *Cardinality*(*activeChairs*) + 1

Temporal properties

$ExistentialLiveness \triangleq$
$\quad \exists\, p \in PLAYERS : \diamond(activePlayers = \{p\})$

$FiniteLiveness \triangleq \diamond\textsc{enabled}\ Sit$

$InfiniteLiveness \triangleq \diamond\square(Cardinality(activePlayers) = 1)$

$Next \triangleq$
$\quad \lor\ Walk$
$\quad \lor\ Sit$
$\quad \lor\ EliminateLoser$
$\quad \lor\ Win$
$\quad \lor\ ChangeMusicPlaying$

$PlayActions \triangleq$
$\quad \lor\ Walk$
$\quad \lor\ Sit$
$\quad \lor\ EliminateLoser$
$\quad \lor\ Win$

$Live \triangleq \mathrm{SF}_{vars}(PlayActions) \land \mathrm{WF}_{vars}(ChangeMusicPlaying)$
$Live \triangleq \textsc{true}$ * don't assume fairness

$Spec \triangleq\ Init \land \square[Next]_{vars} \land Live$ every transition either satisfies the action formula $Next$ or leaves the expression $vars$ unchanged. In particular, this admits "stuttering transitions" that do not affect $vars$. That is to say,
$\square[Next]\_vars \triangleq \square(Next \lor (vars' = vars))$

* Modification History
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