Homework 1
Due Wednesday, January 20

1. In class we will focus exclusively on the theory of second order (continuous) transitions. This problem illustrates how Landau theory can be used to study some first order transitions as well. Let the Landau free energy have the following form:

$$F(M, b, t) = -bM + tM^2 + uM^4 + M^6.$$

Here $b = B/T$ is the reduced (dimensionless) magnetic field, $t = (T - T_c)/T_c$ is the reduced temperature and the coefficient of the M^6 term has been made equal to unity by rescaling the other variables in F.

   (a) In the standard Landau theory of continuous transitions the coefficient u is taken to be positive and the M^6 term is omitted. Assume u can be either positive or negative and analyze the generic form of F in the $u - T$ plane at $B = 0$.

   (b) Show that when $u < 0$ the transition from the ferromagnetic to the paramagnetic state at $B = 0$ is first order, i.e. the magnetization M drops to zero discontinuously at the critical temperature.

   (c) Find the lines of first and second order phase transitions in the $u-T$ plane at $B = 0$. The point in the $u-T$ plane at which the line of first order transitions terminates is called the *tricritical point.* Locate the tricritical point and calculate the critical exponents α, β, γ and δ at this point approaching it along the line $u = 0$.

2. Consider a one-dimensional (1D) classical Ising model:

$$H = -J \sum_{i=0}^{N} \sigma_i \sigma_{i+1},$$

where $\sigma_i = \pm 1$ and N is the total number of sites in the chain.

   (a) Calculate the free energy of this system. *Hint: Start by writing the partition function as a product of N identical 2×2 matrices, each corresponding to a factor $e^{\frac{J}{T}\sigma_i\sigma_{i+1}}$.*

(b) Now generalize the above calculation to include a uniform magnetic B, interacting with the spins. *Hint: The partition function can also be written as a product of N identical 2×2 matrices.* Calculate the macroscopic magnetization per spin as:

$$M = -\frac{1}{N}\frac{\partial F}{\partial B},$$

and show that $M = 0$ for all $T > 0$ and $M = 1$ at $T = 0$.