By a 'map' I mean a morphism in whatever category you're working in.

1 Maps of locally ringed spaces

Let $(X, \mathcal{O}_X)$ and $(Y, \mathcal{O}_Y)$ be locally ringed spaces. A map between them consists of a map $f : X \to Y$ and a map $\hat{f} : \mathcal{O}_Y \to f_*\mathcal{O}_X$ such that the induced map on stalks $\mathcal{O}_{Y,f(p)} \to \mathcal{O}_{X,p}$ is a local.

Where does this map come from? From the sheaf map, we get for free a stalk map

$$(\mathcal{O}_Y)_{f(p)} \to (f_*\mathcal{O}_X)_{f(p)}$$

This latter ring is not in general equal to $(\mathcal{O}_X)_p$ (I will illustrate that with an example in a second). However, there is a 'natural' map

$$(f_*\mathcal{O}_X)_{f(p)} \to (\mathcal{O}_X)_p$$

described as follows. An element in the domain space is a class $[U, g]$ where $f(p) \in U \subseteq Y$ is open and $g \in (f_*\mathcal{O}_X)(U) = \mathcal{O}_X(f^{-1}(U))$. An element in the target space is a pair $[V, h]$ where $p \in V \subseteq X$ is open and $h \in \mathcal{O}_X(V)$. Thus the natural map between them is the map

$$[U, g] \mapsto [f^{-1}(U), g]$$

The 'induced map on stalks' that I mention in my first paragraph is (by definition) the composition of these two maps.

Ex 1. *Consider the map $f : (X = \mathbb{C}) \to \{q\}$ where I'm viewing both sets of algebraic varieties over $\mathbb{C}$. in the usual way. So $\mathcal{O}_X(U) = \{\alpha : U \to k : \alpha$ is regular or a morphism, whatever word you like to use$\}$. Also, $\mathcal{O}_{\{q\}}(\{q\}) = \mathbb{C}$. Then we have that $(f_*\mathcal{O}_X)_{(q=f(3))} = \mathbb{C}$ while $(\mathcal{O}_X)_3 = \mathbb{C}[z]_{(z-3)} \neq \mathbb{C}$. (When I write R_q for a ring R and prime q, I mean what you are calling $(R-q)^{-1}R$.)*

Ex 2. *Let $f : A \to B$ be a ring map, and let $p \in \operatorname{Spec} B$. Then there is an induced map $f_p : A_{f^{-1}(p)} = B_p$ - check that this map is local. This exercise exactly tells you that a ringed space map on affine schemes coming from a ring map is in fact a map of locally ringed spaces.*

Ex 3. *(The most important example). Let X, Y be affine varieties (over your algebraicly closed field k) and let $f : X \to Y$ be a map between them. By their structure sheaves, I mean their structure sheafs as in my first example, ie I'm not talking about schemes right now. Then the map f naturally induces a map $\hat{f} : \mathcal{O}_Y \to f_*\mathcal{O}_X$ (ie you don't need to specify the extra data of the sheaf map - it is induced by the variety map) as follows. For $U \subseteq Y$ open we define $\hat{f}_U$ by*

$$\mathcal{O}_Y(U) \to \mathcal{O}_X(f^{-1}(U))$$

$$(U \to k) \mapsto (f^{-1}(U) \xrightarrow{f} U \to k)$$

This should look familiar - for example, $\hat{f}_Y$ is exactly the $\mathbb{C}$-algebra map $\mathbb{C}[Y] \to \mathbb{C}[X]$ coming from f. The induced map on local rings is exactly the maps you are talking about in your curves course (or will be talking about at some point in that class), which you can check easily to be local maps. The fact that f gave us in a natural way $\hat{f}$ was only because we were talking about such a concrete class of objects, namely varieties. The moral here is that a map of varieties can be naturally viewed as a map of locally ringed spaces - this should be your main example (of maps of locally ringed spaces) until you get more comfortable working with $\operatorname{Spec} \mathbb{C}[X]$.

2 Glueing

Let $X = Y = \mathbb{C}$, with their structure sheaves as varieties making them into locally ringed spaces. Let $U = V = \{z \neq 0\}$ be the same zariski open subset of both X and Y. Then glueing X and Y along the map $U \to V$ (I'm viewing $U \subseteq X$ and $V \subseteq Y$) defined by $p \mapsto \frac{1}{p}$ exactly gives you $\mathbb{CP}^1$ - this has nothing to do with locally ringed spaces.

Now let's describe this in terms of scheme language. Let $X = \operatorname{Spec} \mathbb{C}[x]$ and $Y = \operatorname{Spec} \mathbb{C}[y]$. Let $U \subseteq X$ be the open subset $D(x) = \{(x-a)\}_{a \in \mathbb{C}} \cup \{(0)\}$ and let V be the same set subset of Y (ie replace all the x's with y's). (Note that U may be canonically identified with $\operatorname{Spec} \mathbb{C}[x]_x = \operatorname{Spec} \mathbb{C}[x, x^{-1}] = \operatorname{Spec} \{1, x, x^2, x^3, ...\}^{-1} \mathbb{C}[x]$ (make sure you understand this identification! The last equality is just switching it into your notation for localisation). Now, I want to glue U to V along a certain map - this map should be a scheme map, which means it should come from a ring map. Well, consider the following ring map

$$\phi : \mathbb{C}[y, y^{-1}] \to \mathbb{C}[x, x^{-1}]$$

$$y \mapsto x^{-1}$$

ie $\phi(\sum_i a_i y^i) = \sum_i a_i y^{-i}$. Then we have an induced map

$$\psi : (U = \operatorname{Spec} \mathbb{C}[x, x^{-1}]) \to (V = \operatorname{Spec} \mathbb{C}[y, y^{-1}])$$

Before describing this map, make sure you understand why $U = \{(x-a)\}_{a \in \mathbb{C} - \{0\}} \cup \{(0)\}$. The map ψ, which pulls back a prime ideal along ϕ, is just the map

$$(x - a) \mapsto (y^{-1} - a)$$

$$(0) \mapsto (0)$$

However, notice that $(y^{-1} - a) = (y - a^{-1})$ (these two elements different by a unit - something like $-ya^{-1}$ - so they generate the same ideal. Thus our map ϕ, coming from a ring homomorphism, sends the prime $(x-a)$ (which you should think of as the complex number a) to the prime $(y-a^{-1})$ (which you should think of as the complex number a^{-1}). So the glueing is exactly the same construction from the start of this section, just dressed up with fancy words.

(Technically speaking, I haven't given a scheme map at all - I didn't describe the structure sheaf map. However, as you will see shortly (or have already seen), a ring map $f : A \to B$ determines a map $g : \operatorname{Spec} B \to \operatorname{Spec} A$ AND determines a map $\hat{g} : \mathcal{O}_{\operatorname{Spec} A} \to g_* \mathcal{O}_{\operatorname{Spec} B}$.)