DEFINITION **0.1.** [SPECTRUM] For any ring R we define the Spectrum of A to be the set of prime ideals of A. For the moment, we will consider it as a set but we will put a topology on it soon.

DEFINITION **0.2.** ["FUNCTIONS"] When referring to elements of the Spectrum, if p is a prime ideal we will refer to it as $[p]$ to avoid confusion. For $r \in R$ we refer to r as a "function" on the spectrum which takes as input a prime ideal $[p]$ and its value at $[p]$ will be referred to as $a \mod (p)$.

EXAMPLE **0.3.** Let $R = \mathbb{Z}$ and let $p = \langle 7 \rangle$, then the function 8 on [7] is $1 \mod (7)$.

REMARK **0.4.** An element r lying in the prime ideal $[p]$ corresponds to "a function that is 0 at $[p]$". These functions on Spec(R) are important and will be fundamental in defining a topology on Spec(R). We will want these functions to be as well behaved as possible so we will define the Zariski Topology with that in mind.

EXAMPLE **0.5.** Let $R = \mathbb{Z}$, then since $\mathbb{Z}$ is a UFD, the only prime ideals are (p) and (0) where p is prime. Continuing our discussion of functions, 100 is a function on Spec($\mathbb{Z}$) , its value at [3] is $1 \mod (3)$. Its value at [2] is $0 \mod (2)$. Now consider the "rational" function $\frac{27}{4}$, this has a pole at [2] and a triple root at [3]. Its value at [5] is given by

$$27 \times 4^{-1} \equiv 2 \times (-1) \equiv 3 \mod (5)$$

This will be made more precise later.

EXAMPLE **0.6.** Let $R = \mathbb{R}[x]$. R is a UFD, thus similar to the last example and that of $\mathbb{C}[x]$, one can show that the only prime ideals are $[0], [x-a]$ where $a \in \mathbb{R}$ and $x^2 + ax + b$ where $a, b \in \mathbb{R}$. Note that the nontrivial ideals are maximal, and thus when quotiented by, they yield a field. In particular, one can show that $\mathbb{R}/(x^2 + ax + b) \cong \mathbb{C}$. What does this look like? Well, we have the 0 point, all the real numbers and then for each irreducible quadratic we can interpret it as a pair of conjugate numbers (namely its roots over $\mathbb{C}$). So one can picture the spectrum as the Complex Plane folded over itself on over the real line. In particular, we are "gluing" galois-conjugate points together. So for example $i, -i$ are glued together.

Looking at functions on this space, let $f(x) = x^3 - 1$. Its value at $[x-2]$ is $7 \mod (x-2)$, similarly, its value at $x^2 + 1$ is

$$x^3 - 1 \equiv -x - 1 \mod (x^2 + 1)$$